For what value of a is the vector $\vec{\mathbf{v}} = \langle 5, 8, 3 \rangle$ orthogonal to the vector $\vec{\mathbf{w}} = \langle a, -3, 7 \rangle$?

A. 9

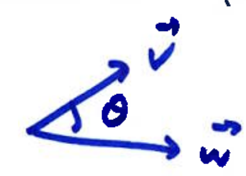
B. $\frac{5}{3}$

C. $\frac{9}{5}$

D. $\frac{1}{9}$

E. $\frac{3}{5}$

angle between vectors



dot product : $\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}| \cos\theta$

orthogonal: $\vec{v} \cdot \vec{w} = 0$

$\vec{v} \cdot \vec{w} = 5a - 24 + 21 = 0$

$5a = 3$

$a = 3/5$

Let $\vec{\mathbf{u}} = \langle 10, 5\rangle$ and $\vec{\mathbf{v}} = \langle 2, 6\rangle$. Find $\underbrace{\operatorname{proj}_{\vec{\mathbf{v}}}(\vec{\mathbf{u}})}_{\text{vector projection}}$, the orthogonal projection of $\vec{\mathbf{u}}$ onto $\vec{\mathbf{v}}$.

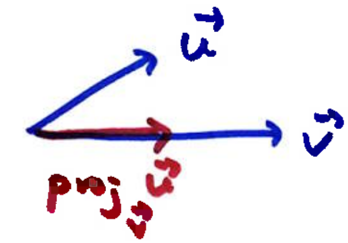
A. $\langle 4, 2\rangle$

B. $\left\langle \frac{1}{2}, \frac{3}{2}\right\rangle$

C. $\langle 5\sqrt{10}, 15\sqrt{10}\rangle$

D. $\left\langle \frac{5}{2}, \frac{15}{2}\right\rangle$

E. $\langle 20\sqrt{5}, 10\sqrt{5}\rangle$



$$\operatorname{proj}_{\vec{v}}\vec{u} = \underbrace{\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}}_{\operatorname{scal}_{\vec{v}}\vec{u}} \frac{\vec{v}}{|\vec{v}|}$$

$$\vec{u}\cdot\vec{v} = 20 + 30 = 50$$

$$|\vec{v}| = \sqrt{2^2 + 6^2} = \sqrt{40}$$

$$\frac{50}{\sqrt{40}} \frac{\langle 2, 6\rangle}{\sqrt{40}} = \frac{50}{40}\langle 2, 6\rangle = \left\langle \frac{5}{2}, \frac{15}{2}\right\rangle$$

Find the area enclosed by the curves $x = 2y - y^2$ and $x = y^2$

parabola opening left or right

y-int: $x = 0$

$0 = 2y - y^2$

$0 = y(2-y)$

$y = 0, \; y = 2$

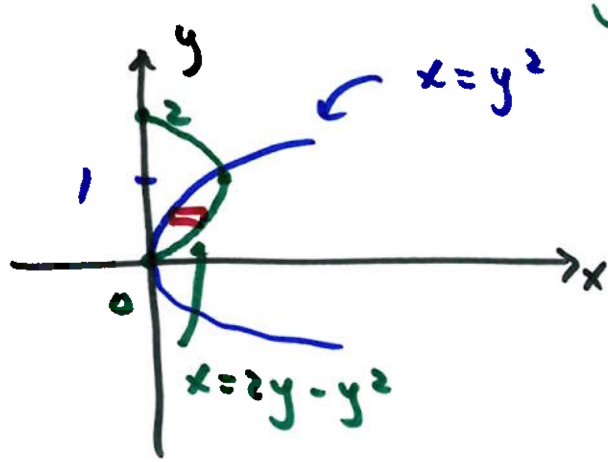
A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. 1

E. $\frac{5}{3}$



integrate the y since equations are x as functions of y

intercepts: $2y - y^2 = y^2$

$2y^2 - 2y = 0 \qquad 2y(y-1) = 0$

$y = 0, \; y = 1$

$$\int_0^1 [\underbrace{(2y-y^2)}_{\text{right}} - \underbrace{(y^2)}_{\text{left}}]\,dy = \int_0^1 2y - 2y^2 \, dy$$

$$= y^2 - \frac{2}{3}y^3 \Big|_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$$

Find a formula for the volume of the following solid: The region bounded by $y = \sin(x)$, the x-axis, $x = 0$ and $x = \frac{\pi}{2}$, is revolved around the line $x = -1$.

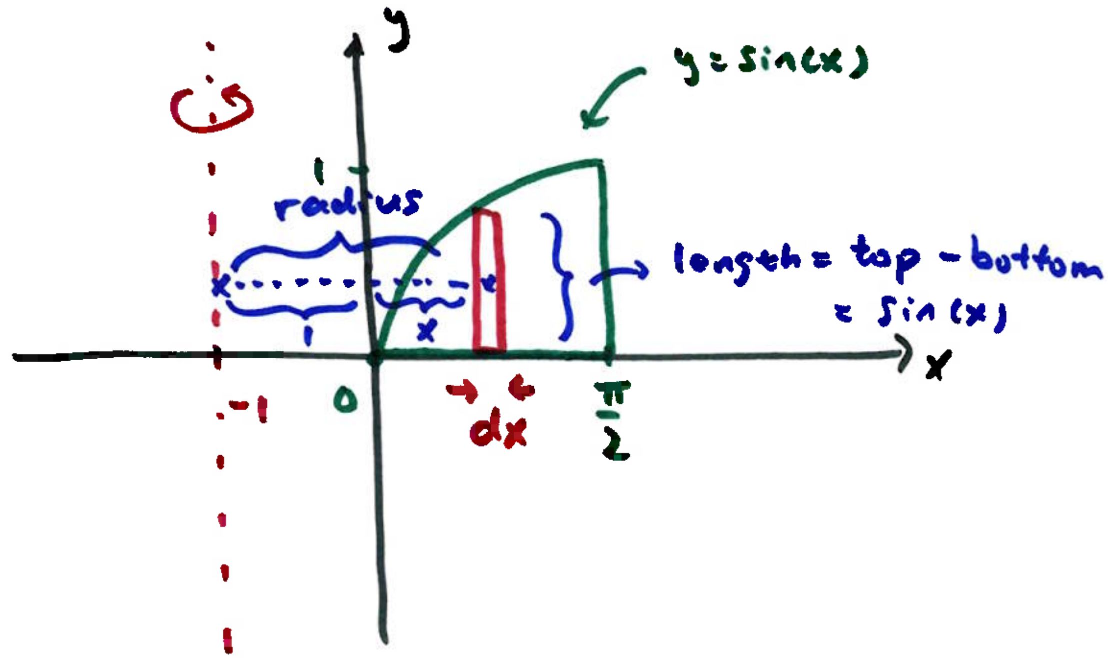
A. $\int_0^{\frac{\pi}{2}} 2\pi(x-1)\sin(x)dx$

B. $\int_0^{1} \pi\left\{[\sin^{-1}(y)]^2 - 1\right\} dy$

C. $\int_0^{1} \pi[\sin^{-1}(y) - 1]^2 dy$

D. $\int_0^{\frac{\pi}{2}} 2\pi(x+1)\sin(x)dx$

E. $\int_0^{\frac{\pi}{2}} 2\pi x \sin(x+1)dx$

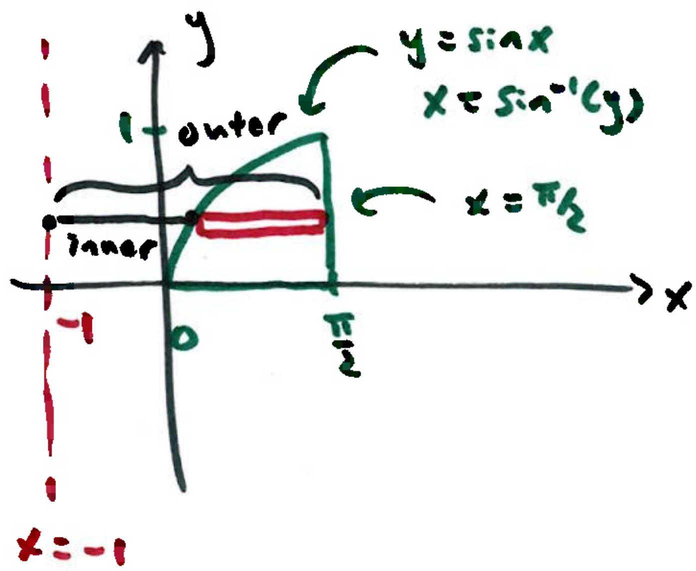


Shell : rectangle parallel to axis of revolution

$$\int_0^{\pi/2} 2\pi \,\underbrace{(\text{radius})}_{1+x}\, \underbrace{(\text{height})}_{\sin(x)}\, \underbrace{(\text{thickness})}_{dx}$$

$$= \int_0^{\pi/2} 2\pi (1+x)\sin(x)\, dx$$

Disk/washer



rectangle perpendicular to axis of rev

$$\int \left[\pi\,(\text{outer radius})^2 - \pi\,(\text{inner})^2\right](\text{thickness})$$

$(\frac{\pi}{2}+1)$ $\quad$ $1+\sin^{-1}(y)$ $\quad$ dy

$$= \int_0^1 \left[\pi\left(\frac{\pi}{2}+1\right)^2 - \pi\left(1+\sin^{-1}(y)\right)^2\right] dy$$

If the method of washers is used, the volume of a solid obtained by revolving a certain region about the x-axis is given by

$$\pi \int_0^1 (x - x^4)\, dx$$

What integral below represents the same volume if the method of cylindrical shells is used?

disk / washers: $\int [\pi (\text{outer})^2 - \pi (\text{inner})^2] (\text{thickness})$

$\pi \int (\text{outer})^2 - (\text{inner})^2 (\text{thickness})$

$\pi \int_0^1 (x - x^4)\, dx$ vertical rectangles

$x = (\text{outer})^2$ $\quad x^4 = (\text{inner})^2$

outer $= \sqrt{x}$ $\quad$ inner $= x^2$

$y = \sqrt{x}$ $\quad y = x^2$

A. $2\pi \int_0^1 (y)(\sqrt{y} - y^2)\, dy$

B. $2\pi \int_0^1 (y)(y^2 - \sqrt{y})\, dy$

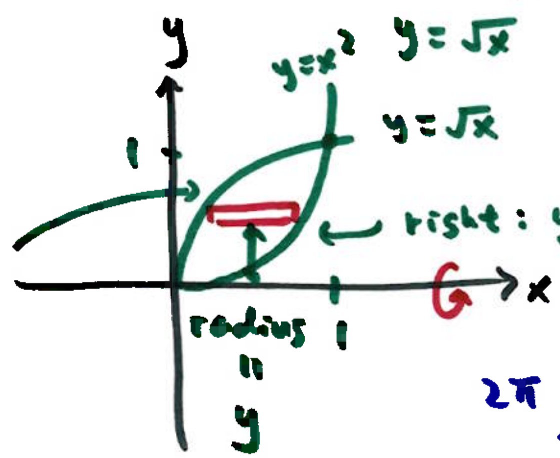
C. $2\pi \int_0^1 (x)(x - x^4)\, dx$

D. $2\pi \int_0^1 (x)(x^4 - x)\, dx$

E. $2\pi \int_0^1 (y)(\sqrt[4]{y} - y)\, dy$

left : $y = \sqrt{x}$

$x = y^2$



right : $y = x^2 \rightarrow x = \sqrt{y}$

$2\pi \int_0^1 (y)(\sqrt{y} - y^2)\, dy$

A swimming pool has a rectangular base that is 5 m long and 6 m wide. The sides are 2 m high and the pool is half full of water. How much work will it take to empty the pool by pumping the water out over the top of the pool? Write your answer in terms of the gravitational acceleration constant g and the density of water ρ.

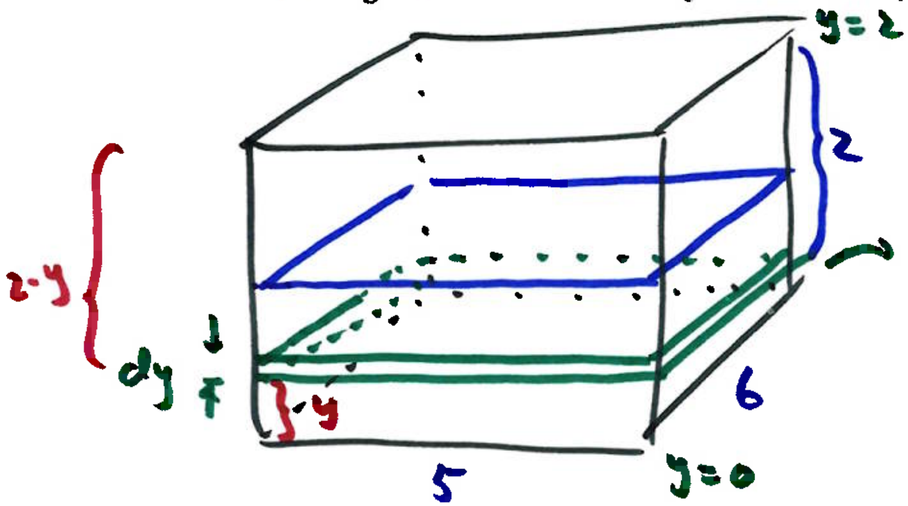
A. $75\rho g$ N·m

B. $90\rho g$ N·m

C. $60\rho g$ N·m

D. $30\rho g$ N·m

E. $45\rho g$ N·m



mass = density · volume

$= \rho \cdot 5 \cdot 6 \cdot dy = 30\rho\, dy$

weight = mass · gravity

$= 30\rho g\, dy$

work = weight · distance

$= 30\rho g(2-y)\,dy$

water surface

total work $= \int_0^1 30\rho g(2-y)\,dy = 30\rho g \int_0^1 2-y\,dy$

bottom of pool

$= 30\rho g\left(2y - \frac{1}{2}y^2\right)\Big|_0^1$

$= 30\rho g \cdot \frac{3}{2} = 45\rho g$