Determine whether the following sequences are convergent or divergent.

(1) $\{a_n = 2n/(3n+1)\}$ C

(2) $\{a_n = \cos n\pi\}$ D

(3) $\{a_n = n\sin(1/n)\}$ C

sequences: $\lim\limits_{n\to\infty} a_n$ exists

A. (1) convergent (2) convergent (3) convergent

B. (1) divergent (2) convergent (3) convergent

C. (1) convergent (2) divergent (3) convergent

D. (1) convergent (2) convergent (3) divergent

E. (1) convergent (2) divergent (3) divergent

(1) $\lim\limits_{n\to\infty} \frac{2n}{3n+1} = \frac{2}{3}$

(2) $1, -1, 1, -1, 1, -1, \ldots$ no limit

(3) $\lim\limits_{n\to\infty} n\sin\left(\frac{1}{n}\right) = \lim\limits_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \to \frac{0}{0}$ $\lim\limits_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right)\cdot \cancel{\frac{-1}{n^2}}}{\cancel{-\frac{1}{n^2}}} = 1$

Test the following series for convergence:

(I) $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$

(II) $\sum_{n=1}^{\infty} \frac{2^n}{n+3^n}$

(III) $\sum_{n=1}^{\infty} \frac{5-2\sqrt{n}}{n^3}$

A. I is divergent; II and III are convergent.

B. I and II are convergent; III is divergent.

C. I and III are divergent; II is convergent.

D. I, II and III are divergent.

E. I, II and III are convergent.

what do these look like?

p-series?

geometric?

(I) $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ when n is large $n > \sqrt{n}$

so $\frac{1}{n+\sqrt{n}} \approx \frac{1}{n}$

compare to $\sum \frac{1}{n}$?

but $\frac{1}{n+\sqrt{n}} \leq \frac{1}{n}$

$\sum \frac{1}{n}$ diverges so direct comparison doesn't work here

limit comparison?

$b_n = \frac{1}{n}$ (known)

$a_n = \frac{1}{n+\sqrt{n}}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n+\sqrt{n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n+\sqrt{n}} = 1$$

so BOTH converge or BOTH diverge

here. $\sum \frac{1}{n}$ diverges we know $\sum \frac{1}{n+\sqrt{n}}$ diverges

(II) $\sum_{n=1}^{\infty} \frac{2^n}{n+3^n}$ as $n\to\infty$ $n+3^n \approx 3^n$

so $\frac{2^n}{n+3^n} \approx \frac{2^n}{3^n}$ convergent geo. series

$\left(\frac{2}{3}\right)^n$ $(r<1)$

direct comp. ok: $\frac{2^n}{n+3^n} \le \frac{2^n}{3^n}$ because larger denominator

so $\sum \frac{2^n}{n+3^n}$ must converge

(III) $\displaystyle\sum_{n=1}^{\infty} \frac{5-2\sqrt{n}}{n^3}$ $\quad n\to\infty \quad \dfrac{5-2\sqrt{n}}{n^3} \approx \dfrac{-2\sqrt{n}}{n^3} \approx \underbrace{\dfrac{-2}{n^{5/2}}}_{\text{conv. p-series } (p>1)}$

direct comp.

$$\frac{5-2\sqrt{n}}{n^3} \geq \frac{-2}{n^{5/2}}$$

finite

or ∞ not useful

lim. comp

$$\lim_{n\to\infty} \frac{\frac{5-2\sqrt{n}}{n^3}}{\frac{-2\sqrt{n}}{n^3}} = \lim_{n\to\infty} \frac{5-2\sqrt{n}}{-2\sqrt{n}} = 1$$

BOTH conv. or BOTH div.

$$\sum_{n=1}^{\infty}\left(\frac{5}{n^3} - \frac{2\sqrt{n}}{n^3}\right)$$

$$= \underbrace{\sum_{n=1}^{\infty} \frac{5}{n^3}}_{\text{conv.}} - \underbrace{\sum_{n=1}^{\infty} \frac{2\sqrt{n}}{n^3}}_{\text{conv.}} = \text{finite}$$

so conv.

so $\sum \dfrac{5-2\sqrt{n}}{n^3}$ must conv.

The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is convergent by the Alternating Series Test. According to the Alternating Series Estimation Theorem what is the smallest number of terms needed to find the sum of the series with error less than 1/15?

A. 1

B. 4

C. 5

D. 2

E. 3

|error| $\leq$ the magnitude of next term (first we throw away)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = \underbrace{-1 + \frac{1}{4} - \frac{1}{9}}_{\text{3 terms}} + \frac{1}{16} - \frac{1}{25} + \cdots$$

first one $< \frac{1}{15}$ so stop at the one before it we can also solve for n: $\frac{1}{n^2} < \frac{1}{15}$

$15 < n^2$

$n^2 > 15$

$n > \sqrt{15}$ → bigger than 3, less than 4

not enough

so $n=4$ is the first time

$\frac{1}{n^2} < \frac{1}{15}$

so we sum up to the one before it → $n=3$

For which values of c is $\sum_{n=1}^{\infty}\left(1+\frac{c^2}{n}\right)^n$ convergent?

A. All values of c.

B. $|c| < 1$.

C. $|c| < 2$.

D. $|c| > 2$.

E. No values of c.

$a_n = \left(1+\frac{c^2}{n}\right)^n$ has power of n

root test looks good here

Root Test : $\lim_{n\to\infty} \sqrt[n]{|a_n|}$ < 1 conv.

$= 1$?

> 1 div.

$$\lim_{n\to\infty} \sqrt[n]{\left(1+\frac{c^2}{n}\right)^n} = \lim_{n\to\infty}\left(1+\frac{c^2}{n}\right) = 1 \quad ?$$

$\lim_{n\to\infty}\left(1+\frac{c^2}{n}\right)^n = e^{\text{something}} \neq 0$ so fails the divergence test geo. $(r)^k$

Find all values of p such that the series $\sum_{k=1}^{\infty} \left(\frac{k^4 + 3k}{k^p + 2} \right)^{1/3}$ converges.

A. $p > 8$

B. $p > 6$

C. $p > 5$

D. $p > 4$

E. $p > 7$

what does it look like as $k \to \infty$

as $k \to \infty$, $\left(\frac{k^4 + 3k}{k^p + 2} \right)^{1/3} \approx \left(\frac{k^4}{k^p} \right)^{1/3}$

$$\approx \left(\frac{1}{k^{p-4}} \right)^{1/3}$$

$$\approx \underbrace{\frac{1}{k^{(p-4)/3}}}_{\text{convergent if}}$$

$\frac{p-4}{3} > 1$

$p - 4 > 3 \quad p > 7$

8. Which of the following series converge conditionally?

~~A~~. $\sum_{n=5}^{\infty} \frac{(-1)^n}{n^{1.1}}$

~~B~~. $\sum_{n=5}^{\infty} \frac{(-1)^n}{n^2-n+1}$

~~C~~. $\sum_{n=5}^{\infty} (-1)^n n$

~~D~~. $\sum_{n=5}^{\infty} (-1)^n e^{-n}$

E. $\sum_{n=5}^{\infty} \frac{(-1)^n}{\ln(\ln n)}$

↳ given $\sum a_k$ converges

but $\sum |a_k|$ diverges

example: $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$ conv.

$\sum_{k=1} \frac{1}{k}$ div.

A. $\sum \frac{(-1)^n}{n^{1.1}}$ conv. because $\lim_{n\to\infty} \frac{1}{n^{1.1}} = 0$ and $\frac{1}{n^{1.1}}$ is nonincreasing

$\sum \frac{1}{n^{1.1}}$ conv. so A. conv. absolutely

B. $\sum \frac{(-1)^n}{n^2-n+1}$ looks $\sum \frac{(-1)^n}{n^2}$ as $n \to \infty$

conv. by Alt. series test

conv. p-series after taking absolute value

C. $\lim_{n\to\infty} (-1)^n n \neq 0$ doesn't converge

D. $\sum (-1)^n e^{-n} = \sum \frac{(-1)^n}{e^n}$ conv.? yes, by alt. series test

$\sum \frac{1}{e^n}$ conv? yes

$= \sum \left(\frac{1}{e}\right)^n$ geo. series w/ $r = \frac{1}{e} < 1$

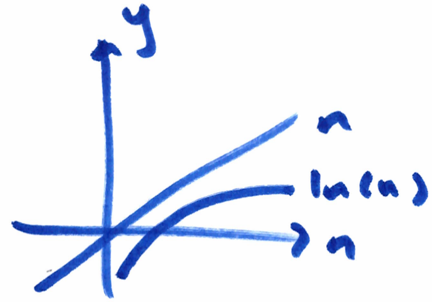
conv. absolutely

E. $\sum \frac{(-1)^n}{\ln(\ln(n))}$ conv. by alt. series test

$\sum \frac{1}{\ln(\ln(n))}$ conv? No.

$\ln(n) < n$

$\ln(\ln(n)) < \ln(n)$



$$\frac{1}{\ln(\ln(n))} > \frac{1}{\ln(n)} > \frac{1}{n}$$

$$\sum \frac{1}{\ln(\ln(n))} > \sum \frac{1}{\ln(n)} > \underbrace{\sum \frac{1}{n}}_{\infty}$$

div.

so $\sum \frac{1}{\ln(\ln(n))}$ conv. conditionally