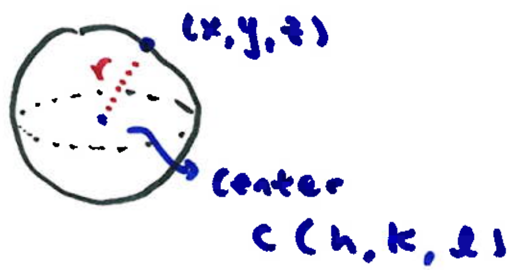
13.2 Vectors in 3D (part 2)

vectors can be used to build shapes

for example, sphere



vector from center $c(h,k,\ell)$
to point on sphere (x,y,z)
must have a magnitude of r (radius)

vector from center to point: $\langle x-h,\ y-k,\ z-\ell \rangle$

$$|\langle x-h,\ y-k,\ z-\ell \rangle| = r$$

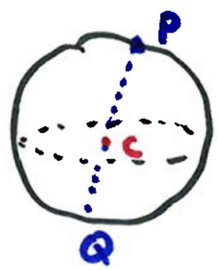
$$\sqrt{(x-h)^2+(y-k)^2+(z-\ell)^2} = r$$

$$\boxed{(x-h)^2+(y-k)^2+(z-\ell)^2 = r^2}$$

standard Form of eq. of sphere

example Find eq. of sphere whose diameter has endpoints

$P(-2, 3, 6)$, $Q(4, -7, 5)$



need center, radius

center: midpoint between P, Q

average of x, y, z coordinates

$$C\left(\frac{-2+4}{2}, \frac{3-7}{2}, \frac{6+5}{2}\right)$$

$$= C\left(1, -2, \frac{11}{2}\right)$$

radius: half distance of $\vec{PQ}$

$$\vec{PQ} = \langle 6, -10, -1 \rangle$$

$$|\vec{PQ}| = \sqrt{36+100+1} = \sqrt{137} \qquad \text{radius} = \frac{\sqrt{137}}{2}$$

eq:

$$\boxed{(x-1)^2 + (y+2)^2 + \left(z - \frac{11}{2}\right)^2 = \left(\frac{\sqrt{137}}{2}\right)^2 = \frac{139}{4}}$$

example Find center and radius of

$$x^2+y^2+z^2-14x+16y-10z+102=0$$

not in standard form, let's turn it into the standard form

complete the square:

$$x^2\underbrace{-14}_{\text{half of this squared}}x+\underline{49}+y^2+\underbrace{16}_{\text{half of this squared}}y+\underline{64}+z^2-10z+\underline{25}=-102+49+64+25$$

$$\underbrace{x^2-14x+49}+y^2+16y+64+z^2-10z+25=36$$

$$(x-7)^2+(y+8)^2+(z-5)^2=6^2$$

center: $(7,-8,5)$

radius: 6

complete the square: coefficient of squared terms must be 1

$(x-h)^2+(y-k)^2+(z-\ell)^2=r^2$

all vectors from center to surface have length r

what about $(x-h)^2+(y-k)^2+(z-\ell)^2 \le r^2$

all vectors from center have length $\le r$

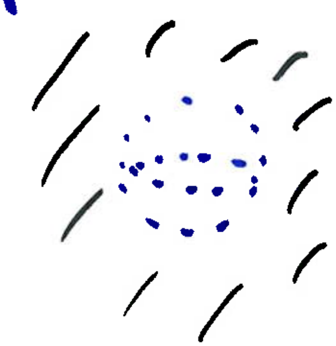
Sphere + all space inside $\rightarrow$ solid ball

what about $(x-h)^2+(y-k)^2+(z-\ell)^2 < r^2$

ball w/o the spherical "skin"

$(x-h)^2+(y-k)^2+(z-\ell)^2 > r^2$

vectors have lengths $> r$



Space outside NOT including Sphere

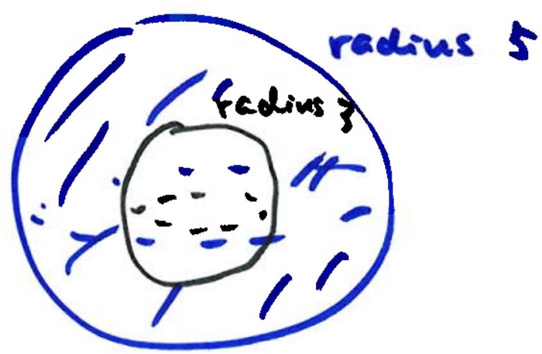
$$9 \le (x-1)^2 + (y+2)^2 + (z-5)^2 \le 25$$

$\downarrow$ radius 3

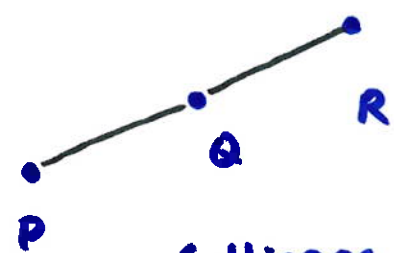
center: $(1, -2, 5)$

$\downarrow$ radius 5

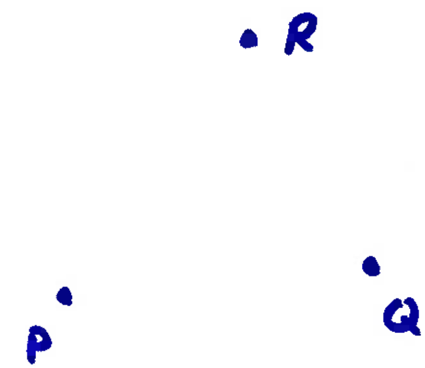
vectors from center have magnitudes between 3 and 5



given 3 points in 3D space ($\mathbb{R}^3$), how do we know if they lie along a line? (collinear)

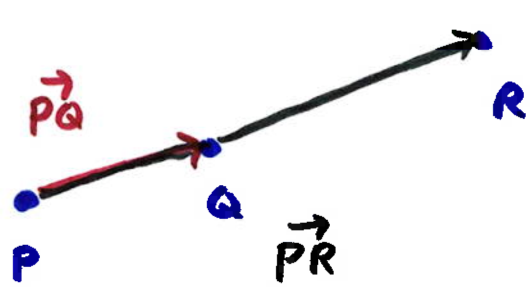


collinear



not collinear

note: distances between them are not relevant for collinear



if collinear, $\vec{PQ} \parallel \vec{PR}$

or equivalently, $\vec{PQ} = \underbrace{k}_{\text{some constant}} \vec{PR}$

example $P(1, 3, 2)$, $Q(5, 1, 3)$, $R(x, 2, z)$ are collinear

find x, z

$\vec{PQ} = \langle 4, -2, 1 \rangle$

$\vec{PR} = \langle x-1, -1, z-2 \rangle$

$\vec{PQ} \parallel \vec{PR} \longrightarrow \vec{PQ} = K\,\vec{PR}$

$\langle 4, -2, 1 \rangle = K \langle x-1, -1, z-2 \rangle$ vector equation

turns into scalar equations by equating components

$x: \; 4 = K(x-1)$

$y: \; -2 = K(-1) \longrightarrow K = 2$ now use this in the other equations to find x, z

$z: \; 1 = K(z-2)$

$x: \; 4 = 2(x-1) \longrightarrow \boxed{x = 3}$

$z: \; 1 = 2(z-2) \longrightarrow \boxed{z = \frac{5}{2}}$