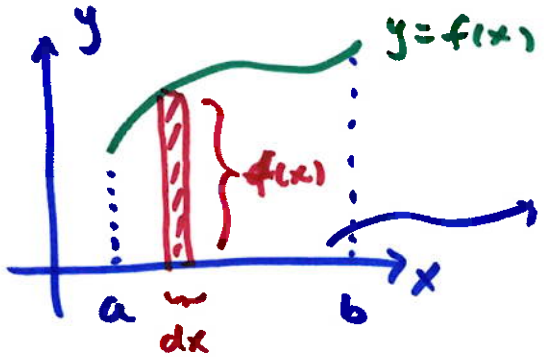
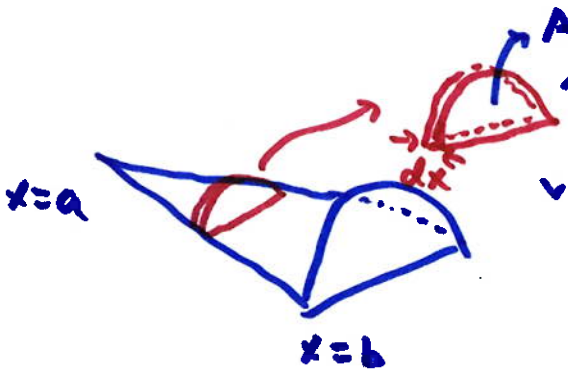
6.3 Volumes by Slicing



$$\text{area} = \int_a^b f(x)\,dx$$

$f(x)\,dx$ → thin rectangle area at x

→ accumulates all from $x=a$ to $x=b$



$A(x)$ → volume of this is area at x times thickness dx

volume of this?

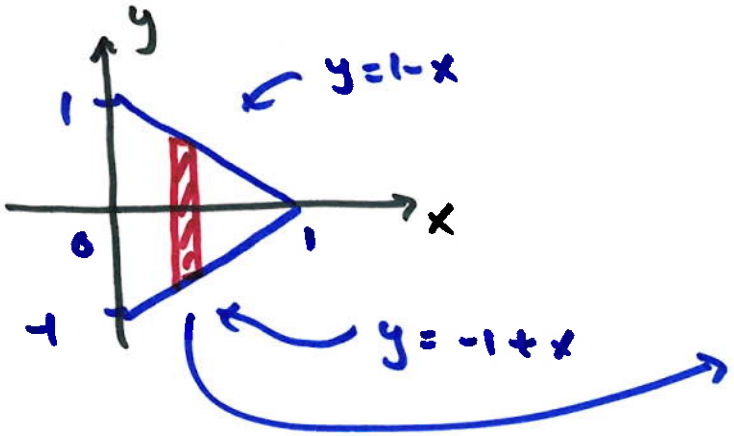
$$\text{volume} = \int_a^b A(x)\,dx$$

dx → thickness of slice

$A(x)$ → area at x

example the base of the object is region bounded by $y = 1 - x$ and $y = -1 + x$ from $x = 0$ to $x = 1$

each slice perpendicular to xy-plane is a semicircle



↳ thickness is dx

half a circle : area $= \frac{1}{2}\pi (\text{radius})^2$

the shaded bar is the diameter of this semicircle

diameter = length of shaded bar $= (1-x) - (-1+x) = 2 - 2x$

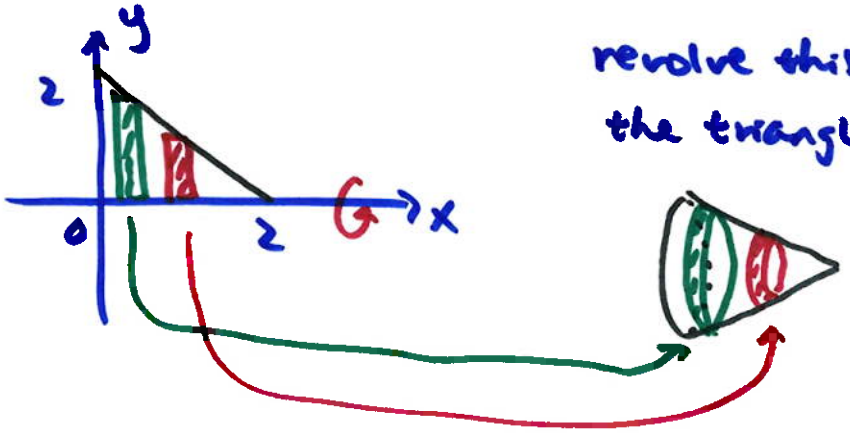
radius $= 1 - x$

the volume of each slice $= \frac{1}{2}\pi (1-x)^2 dx$

stack all possible slices : $\int_0^1 \frac{1}{2}\pi (1-x)^2 dx = \ldots = \boxed{\frac{\pi}{6}}$

volume of

Volume of revolution

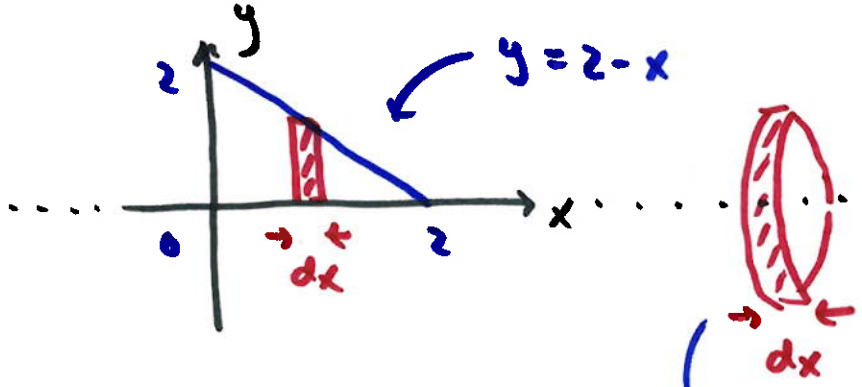


revolve this region about x-axis

the triangle sweeps out a cone

volume ?

find the volume of each disk at some x, then integrate to accumulate



radius = height of bar = $2 - x$

$y = 0$

volume = (area)(thickness) = $\pi (\text{radius})^2 \, dx$

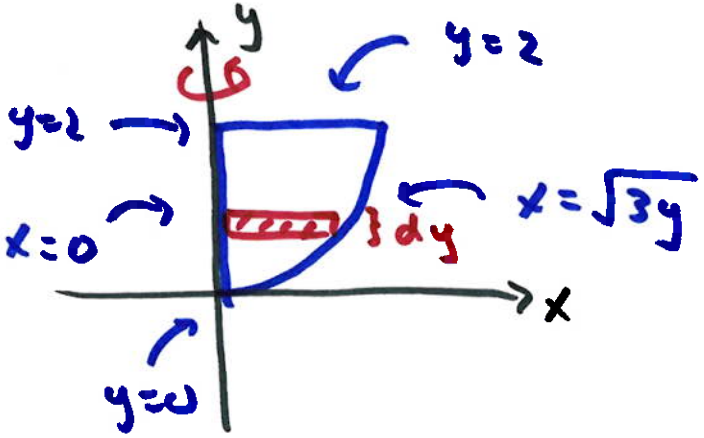
$= \pi (2-x)^2 dx$

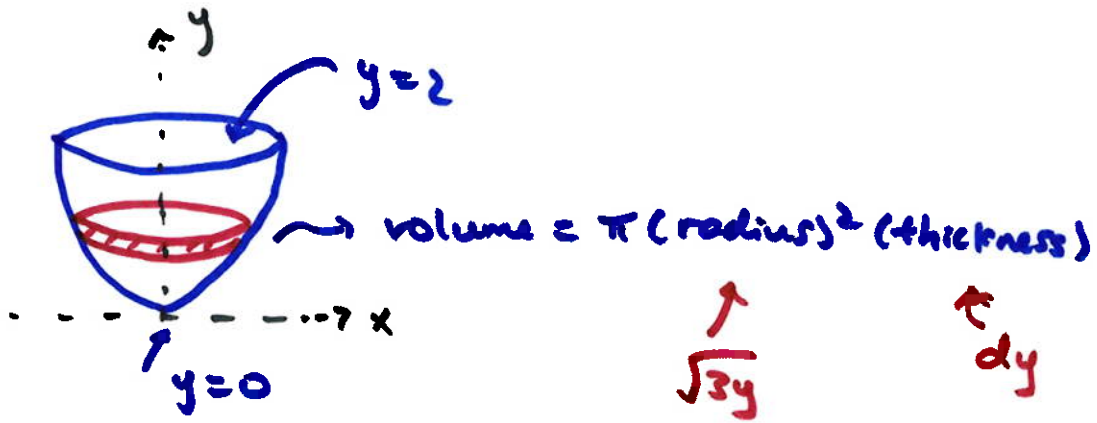
this is one slice starting at $x=0$, ending at $x=2$

integrate to accumulate:

$$\int_0^2 \pi (2-x)^2 dx = \cdots = \boxed{\frac{8\pi}{3}}$$

Example volume of solid obtained by revolving the region bounded by $x=\sqrt{3y}$, $x=0$, $y=2$, about the y-axis





all these y's and dy → integrate in terms of y

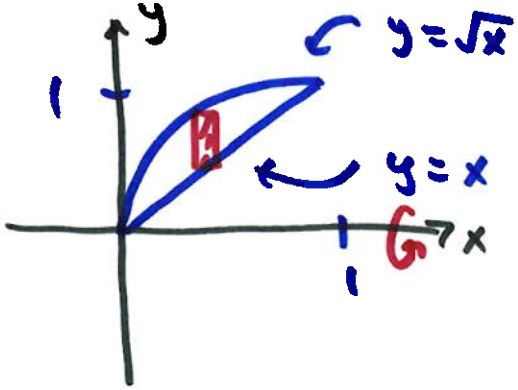
volume of one slice $= \pi(\sqrt{3y})^2\,dy$

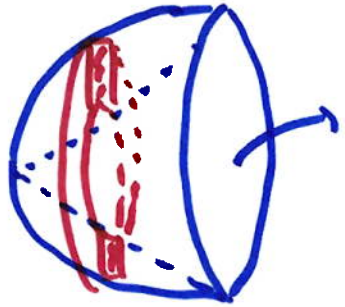
accumulate from $y=0$ to $y=2$

$$\int_0^2 \pi(\sqrt{3y})^2\,dy = \int_0^2 3\pi y\,dy = 3\pi\,\frac{y^2}{2}\Big|_0^2 = \boxed{6\pi}$$

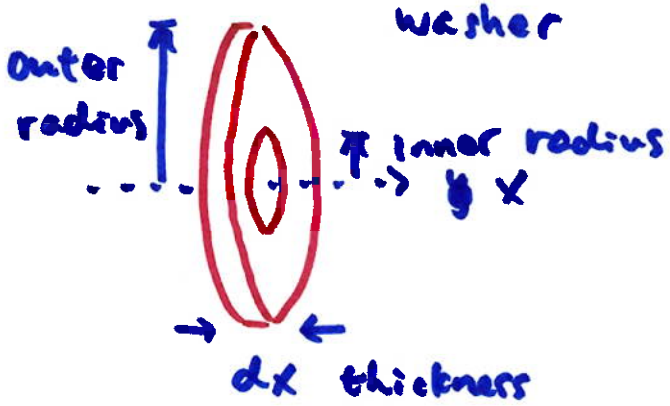
this method, accumulate disks (or washers) is called the **Disk / Washer Method**

example Solid obtained by revolving the region bounded by $y=\sqrt{x}$, $y=x$, about x-axis





hollow cone space



washer

area = π area of big − area of small

$= \pi(\text{outer})^2 - \pi(\text{inner})^2$

volume of washer $= \left[\pi(\text{outer})^2 - \pi(\text{inner})^2\right]dx = \left(\pi(\sqrt{x})^2 - \pi(x)^2\right)dx$

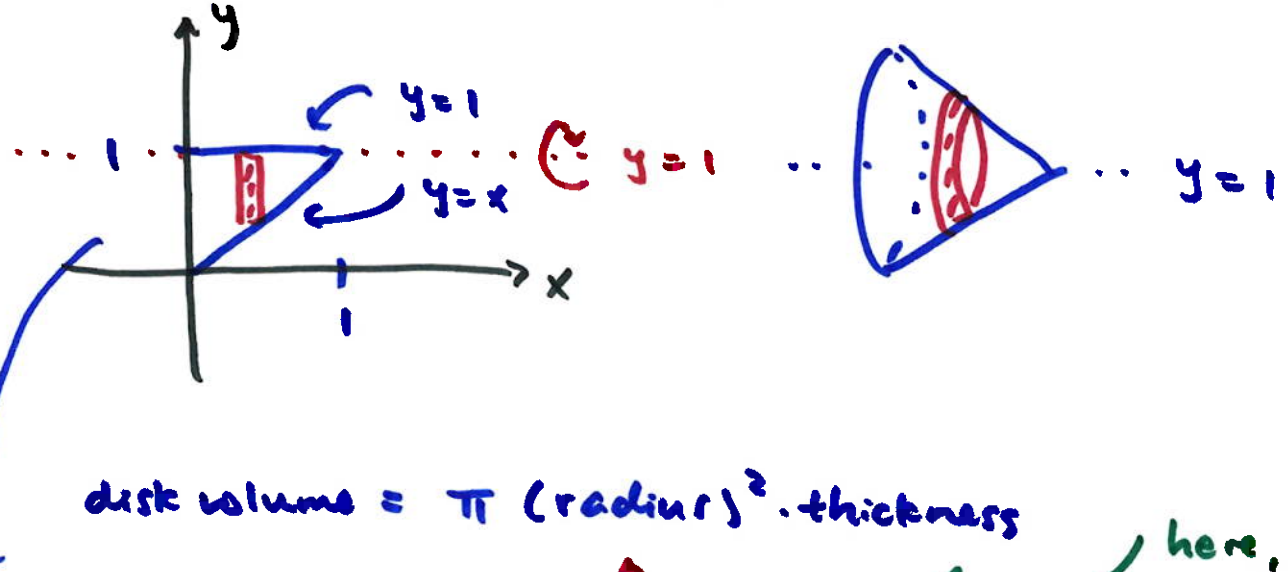
start: $x=0$

end: $x=1$

volume of whole thing: $\int_0^1 \left[\pi(\sqrt{x})^2 - \pi(x^2)\right]dx = \cdots = \boxed{\frac{\pi}{6}}$

Example region is bounded by $y=1$, $y=x$, $x=0$

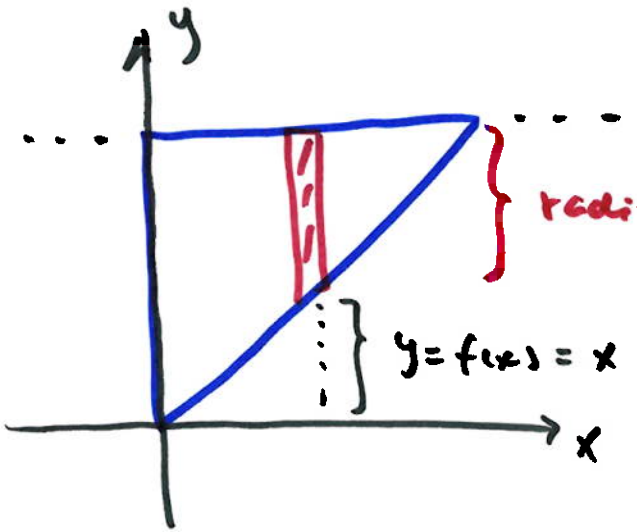
not revolved around $y=1$



disk volume = π (radius)^2 · thickness

here, dx

measured from axis of revolution



$$\int_0^1 \pi (1-x)^2 \, dx$$

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