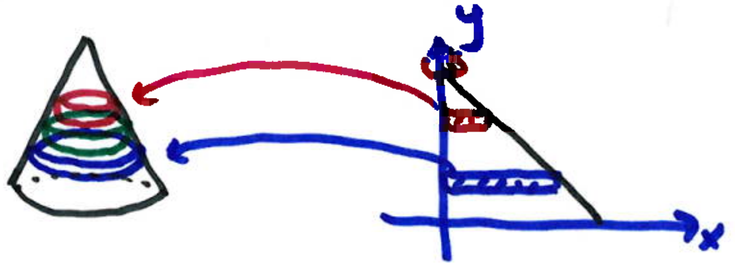
6.4 Volumes by Shells

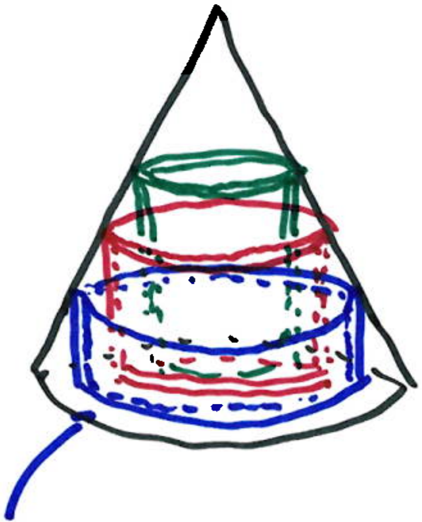
last time: method of disk/washer

stack volumes of thin disks



disk/washer: rectangles perpendicular to axis of revolution

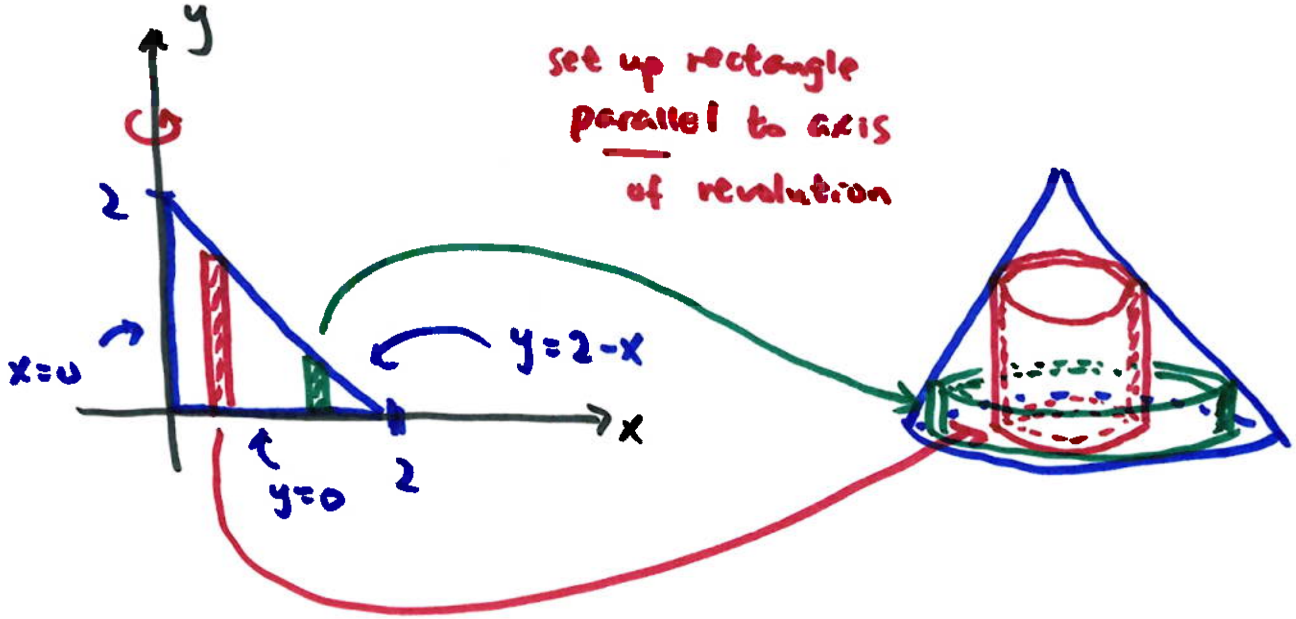
today: method of cylindrical shells



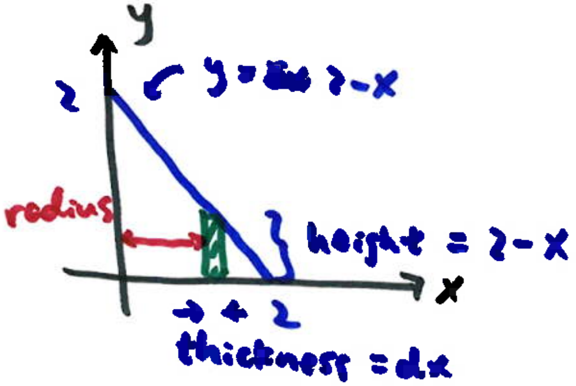
thin shell (think of onion layers)

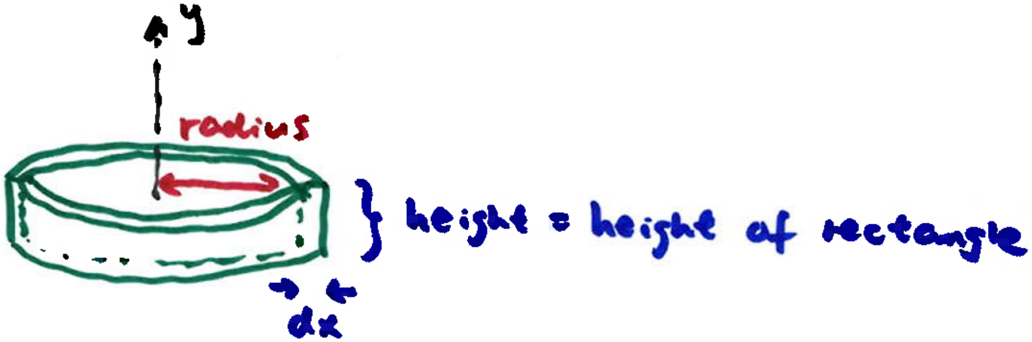
stacking infinitely-many shells to form, in this case, a cone

example Use shells to find volume of solid obtained by revolving the region bounded by $y=2-x$, $y=0$, $x=0$, revolved around y-axis.


set up rectangle <u>parallel</u> to axis of revolution

calculate volume of one shell, then integrate to accumulate all shells

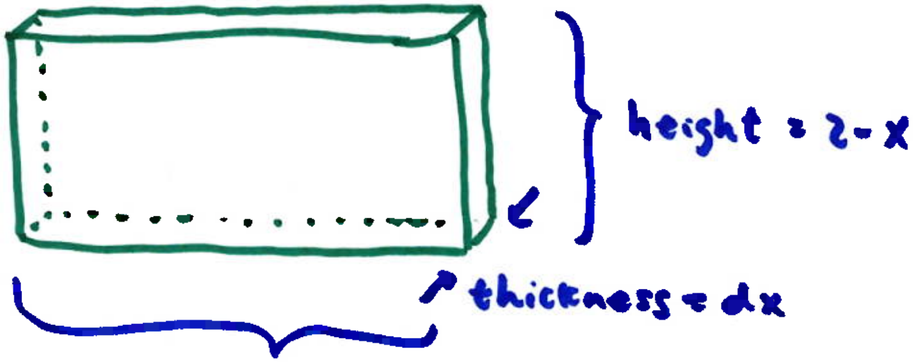




radius = how far rectangle is from axis of revolution

here, radius = x

volume of green shell : unwrap it



length = circumference

= $2\pi \cdot$ radius

= $2\pi \cdot x$

volume = $(2\pi x)(2-x)(dx)$ = $2\pi x(2-x)dx$

this is one very thin rectangle / shell

we want to accumulate ALL such shells

start at $x=0$, end at $x=2$

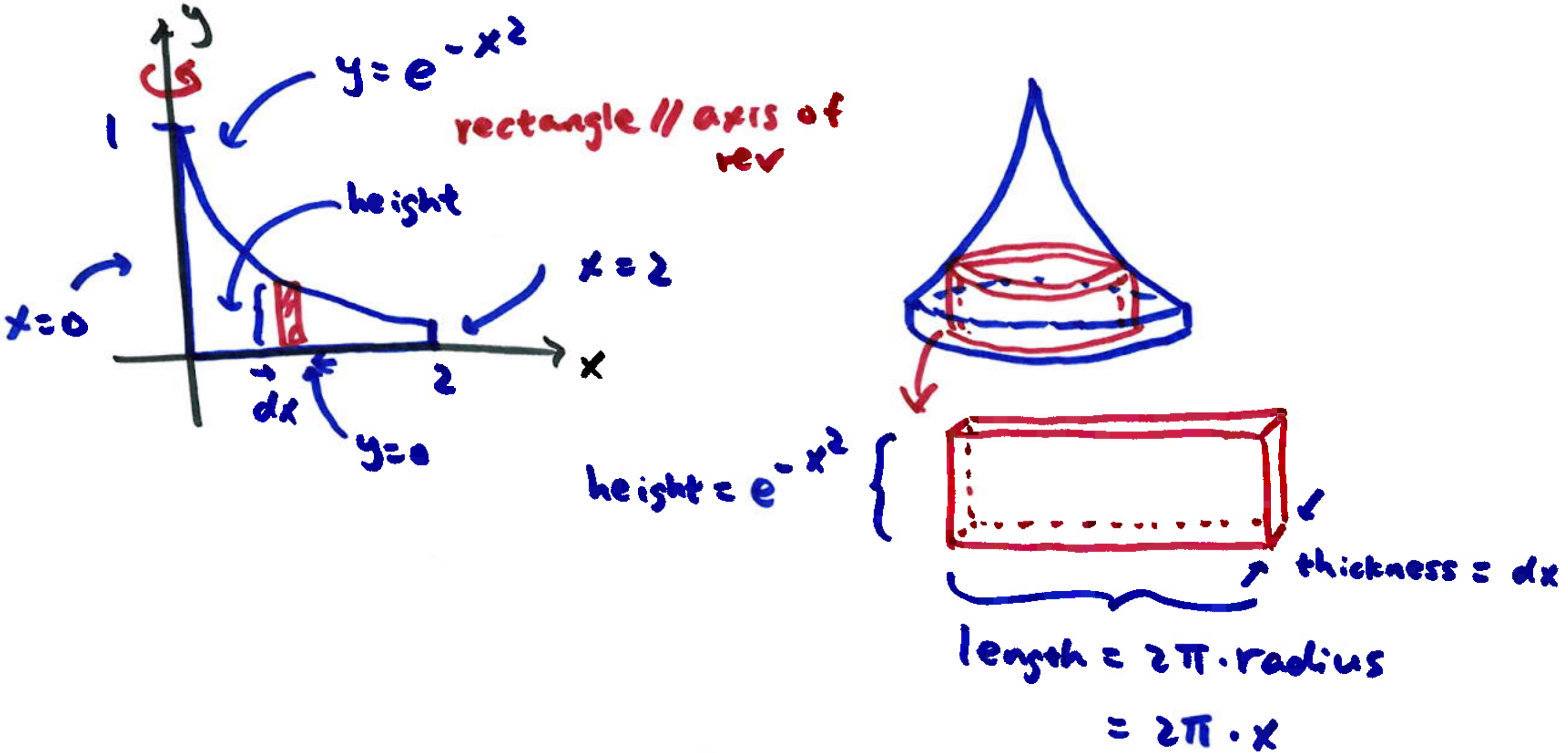
volume of the resulting cone

right end of region

$$= \int_0^2 \underbrace{2\pi x(2-x)\,dx}_{\text{one shell}} = \dots = \boxed{\frac{8\pi}{3}}$$

left end of region

Example Region bounded by $y=e^{-x^2}$, $x=0$, $y=0$, $x=2$
revolved around y-axis



volume of one shell: $2\pi x e^{-x^2} dx$

volume of object: sum all thin shell volumes by integration

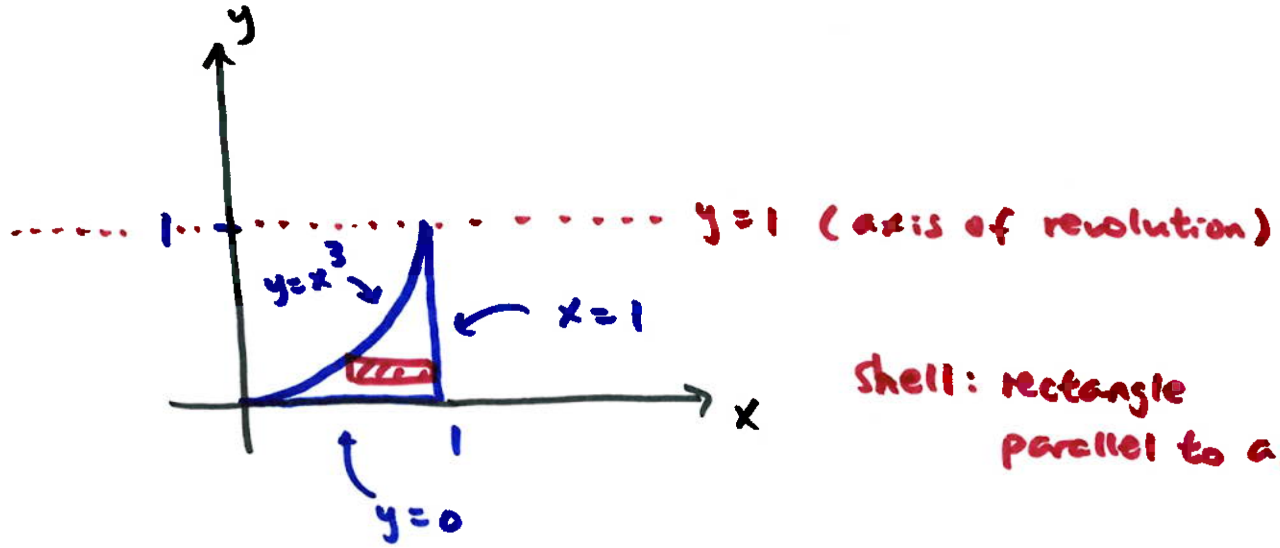
$$\int_0^2 \underbrace{2\pi x e^{-x^2} dx}_{\text{one shell}} = \text{by subs } \overset{}{\underset{u=-x^2}{\cancel{y=e^{x^2}}}} \text{ etc} = \dots = \boxed{\pi(1-e^{-4})}$$

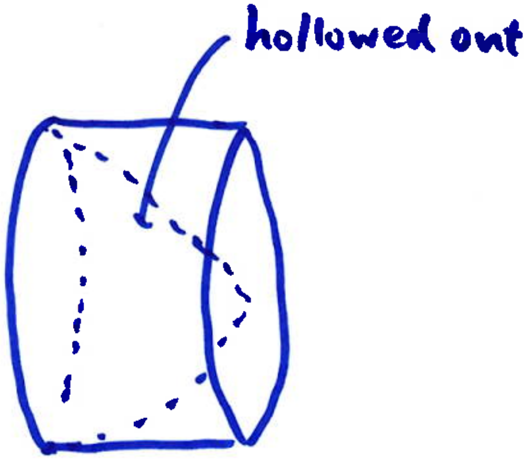
← right end of region

← left end of region

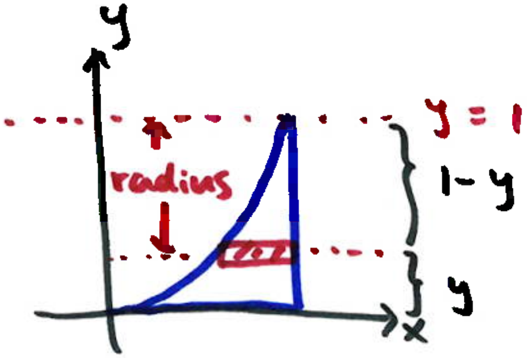
example Region bounded by $y = x^3$, $x=1$, $y=0$

revolved around $y=1$





Shell: rectangle
parallel to axis of rev



radius = distance of rectangle from axis of rev.

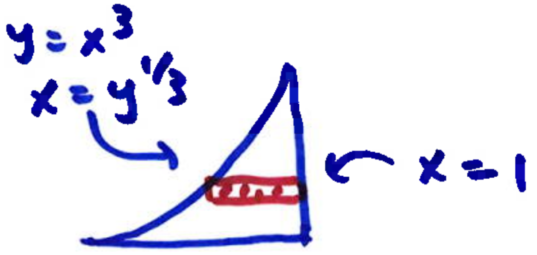
$= 1-y$

thickness $= dy$ ← integrate in terms of y
NO x in any part

"height" $= 1 - y^{1/3}$

right curve
$(x=1)$

left curve
$(y=x^3$
but turned
into $x=y^{1/3})$



volume of one shell $= 2\pi \underset{\text{radius}}{(1-y)} \underset{\text{"height"}}{(1-y^{1/3})} \underset{\text{thickness}}{dy}$

accumulate ALL by integration from $\underset{\text{(bottom of region)}}{y=0}$ to $\underset{\text{(top of region)}}{y=1}$

$$\int_0^1 2\pi (1-y)(1-y^{1/3})\, dy = \cdots = \boxed{\frac{5\pi}{14}}$$