The tangent plane to the graph of the surface $z = e^{2x} \ln y$ at the point $(1, 1, 0)$ is

normal vector

point ✓

A. $e^2 y - z = e^2$

B. $2e^2 y - z = 1$

C. $x - e^2 y + z = 1$

D. $2e^2 x - e^2 y = e^2$

E. $2e^2 x - e^2 y + z = e^2$

let $F = e^{2x} \ln y - z$

$\vec{\nabla} F$ is normal to the surface $z = e^{2x} \ln y$

$$\vec{\nabla} F = \left\langle \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right\rangle = \left\langle 2e^{2x} \ln y, \frac{e^{2x}}{y}, -1 \right\rangle$$

at $(1,1,0)$ $\quad \vec{\nabla} F = \langle 0, e^2, -1 \rangle$

plane: $\quad 0(x-1) + e^2(y-1) - 1(z-0) = 0$

$$e^2 y - e^2 - z = 0$$

Find the maximum value of the function $f(x, y) = 8x - 6y$ subject to the constraint $(x-1)^2 + y^2 = 1$

A. 18

B. 19

C. -2

D. 10

E. 11/5

Lagrange multipliers

$g(x,y) = (x-1)^2 + y^2 - 1 = 0$

$f(x,y) = 8x - 6y$

Solve $\vec{\nabla} f = \lambda \vec{\nabla} g$ for x, y then find max of $f(x,y)$

$\vec{\nabla} f = \langle 8, -6 \rangle$ $\quad \vec{\nabla} g = \langle 2(x-1), 2y \rangle$

$\langle 8, -6 \rangle = \lambda \langle 2(x-1), 2y \rangle$

$8 = \lambda \cdot 2(x-1) \rightarrow \lambda = \frac{8}{2(x-1)} \rightarrow \lambda = \frac{4}{x-1}$

$-6 = \lambda \cdot 2y \rightarrow \lambda = \frac{-3}{y}$

$\frac{4}{x-1} = \frac{-3}{y}$

$x - 1 = -\frac{4}{3} y$

$x = 1 - \frac{4}{3} y$

$(x-1)^2 + y^2 = 1$

$\frac{16}{9} y^2 + y^2 = 1$

$\frac{25}{9} y^2 = 1$

$y^2 = \frac{9}{25}$ $\qquad y = \frac{3}{5}, \; -\frac{3}{5}$

$x = 1 - \frac{4}{3}y$ $\qquad x = \frac{1}{5}, \; \frac{9}{5}$

points to check: $\left(\frac{9}{5}, -\frac{3}{5}\right), \left(\frac{1}{5}, \frac{3}{5}\right)$

$f\left(\frac{9}{5}, -\frac{3}{5}\right) = 18 \leftarrow$ max

$f\left(\frac{1}{5}, \frac{3}{5}\right) =$

The function $f(x, y) = x^3 - y^3 - 3xy + 6$ has local extrema consisting of:

A. One local maximum and one local minimum.

B. One local maximum and one saddle point.

C. One local minimum and one saddle point.

D. One local maximum, one local minimum, and one saddle point.

E. One local minimum and two saddle points.

critical points: $f_x = 0$ and $f_y = 0$

$f_x = 3x^2 - 3y = 0 \rightarrow x^2 = y$

$f_y = -3y^2 - 3x = 0 \rightarrow \underline{x = -y^2}$

$(-y^2)^2 = y$

$y^4 = y \quad y^4 - y = 0 \quad y(y^3 - 1) = 0$

$y = 0, \quad y = 1$

$x = 0, \quad x = -1$

critical pts: $(0, 0), (-1, 1)$ $\quad D = f_{xx} f_{yy} - (f_{xy})^2$

$f_{xx} = 6x \quad f_{yy} = -6y \quad f_{xy} = -3$

$D = -36xy - 9$

$D(0,0) < 0 \rightarrow$ saddle pt

$D(-1,1) > 0, \quad f_{xx}(-1,1) < 0 \rightarrow$ max

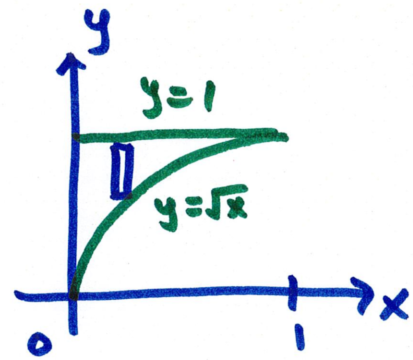
Change the order of integration and evaluate

$$\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx$$

$$\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx$$

$0 \le x \le 1$

$\sqrt{x} \le y \le 1$



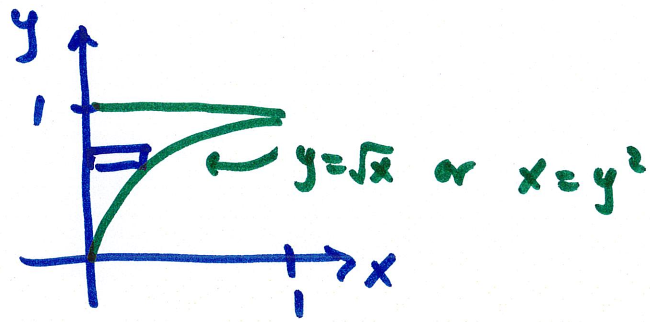
A. $\frac{1}{2}e$

B. $\frac{1}{2}(e-1)$

C. $\frac{1}{3}e$

D. $\frac{1}{3}(e-1)$

E. e



$0 \le y \le 1$

$0 \le x \le y^2$

$$\int_0^1 \int_0^{y^2} e^{y^3}\, dx\, dy$$

$\cancel{u = y^3}$

$\cancel{du = 3y^2 d}$

$$= \int_0^1 x\Big|_0^{y^2} e^{y^3}\, dy$$

$$= \int_0^1 y^2 e^{y^3}\, dy \qquad u = y^3, \quad du = 3y^2\,dy$$

$$\int_0^1 \tfrac{1}{3} e^u\, du = \tfrac{1}{3} e^u \Big|_0^1 = \tfrac{1}{3}e^1 - \tfrac{1}{3}e^0$$

$$= \tfrac{1}{3}e - \tfrac{1}{3}$$

Do NOT evaluate. Rewrite the integral in cylindrical coordinates.

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} \boxed{\sqrt{x^2+y^2}}\, dz\, dx\, dy$$

A. $\int_0^{\pi/2} \int_0^2 \int_r^{\sqrt{8-r^2}} r^2\, dz\, dr\, d\theta$

B. $\int_0^{\pi/2} \int_0^2 \int_r^{\sqrt{8-r^2}} r\, dz\, dr\, d\theta$

C. $\int_0^{\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} r^2\, dz\, dr\, d\theta$

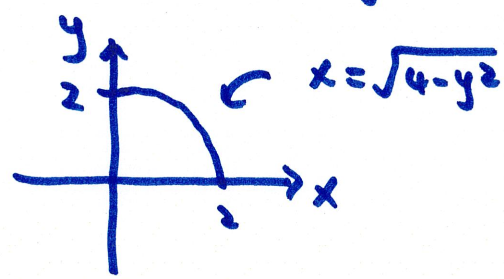
D. $\int_0^{\pi} \int_0^2 \int_r^{\sqrt{8-r^2}} r\, dz\, dr\, d\theta$

E. None of the above.

floor : xy-plane

$0 \leq y \leq 2$

$0 \leq x \leq \sqrt{4-y^2}$



polar equivalent : $0 \leq \theta \leq \frac{\pi}{2}$

$0 \leq r \leq 2$

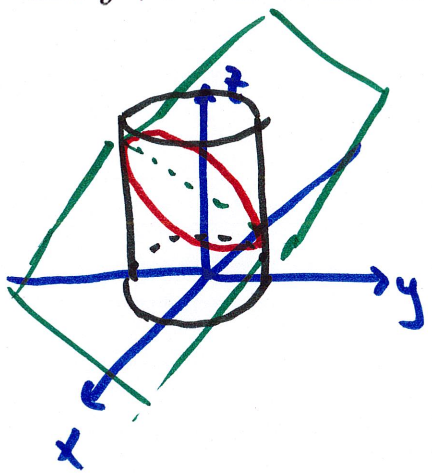
$x^2+y^2 = r^2$

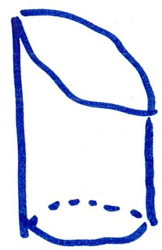
$\sqrt{x^2+y^2} \leq z \leq \sqrt{8-x^2-y^2}$

$\underbrace{\sqrt{r^2}}_{= r} \leq z \leq \sqrt{8-r^2}$

new integral: $\int_0^{\pi/2} \int_0^2 \int_r^{\sqrt{8-r^2}} \underbrace{r}_{\sqrt{x^2+y^2}}\ \overbrace{r\,dz\,dr\,d\theta}^{dV}$

Let E be the solid region enclosed by the cylinder $x^2 + y^2 = 1$, and the planes $z = 0$ and $y + z = 2$. Which of the following triple integrals is equal to the volume of E?





volume = ?

A. $\displaystyle\int_0^{2\pi}\int_0^{1}\int_0^{2-r\sin\theta} r\,dz\,dr\,d\theta$

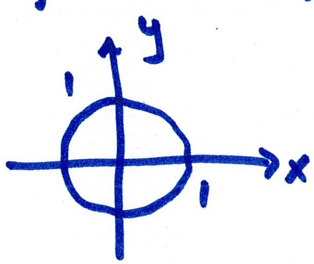
B. $\displaystyle\int_0^{2\pi}\int_0^{1}\int_0^{2-\sin\theta} r\,dz\,dr\,d\theta$

C. $\displaystyle\int_0^{\pi}\int_0^{1}\int_0^{2-r\sin\theta} r\,dz\,dr\,d\theta$

D. $\displaystyle\int_0^{\pi}\int_0^{1}\int_0^{2-\sin\theta} r\,dz\,dr\,d\theta$

E. $\displaystyle\int_0^{2\pi}\int_0^{\sin\theta}\int_0^{2} r\,dz\,dr\,d\theta$

projection onto xy-plane



$0 \le r \le 1$

$0 \le \theta \le 2\pi$

$0 \le z \le 2 - y$ — from plane $y + z = 2$

$y = r\sin\theta$

$$\int_0^{2\pi}\int_0^{1}\int_0^{2-r\sin\theta} r\,dz\,dr\,d\theta$$

Find the volume of the solid that is enclosed by $\underbrace{x^2+y^2+z^2=1}_{\text{sphere radius 1 center origin}}$, $\overbrace{x^2+y^2+z^2=4}^{\text{sphere radius 2 center origin}}$, and $\underbrace{z=\sqrt{x^2+y^2}}$. cone

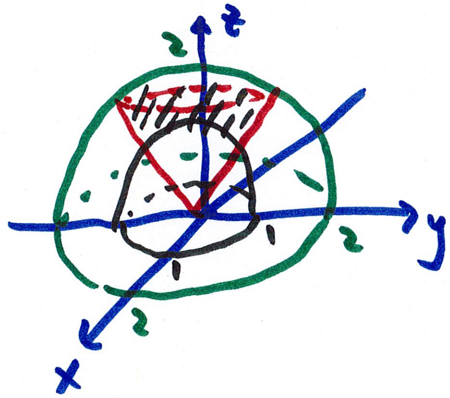
A. $\dfrac{14\pi}{3}\left(1-\dfrac{\sqrt{2}}{2}\right)$

B. $\dfrac{28\pi}{3}$

C. $\dfrac{14\pi}{3}\left(1+\dfrac{\sqrt{2}}{2}\right)$

D. $3\pi\left(1-\dfrac{\sqrt{2}}{2}\right)$

E. 3π



Spherical is good

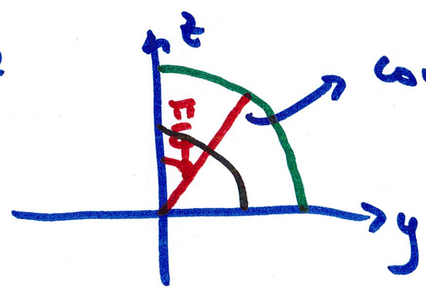
$1 \le \rho \le 2$

$0 \le \theta \le 2\pi$ (all around z-axis)

$0 \le \phi \le \frac{\pi}{4}$

z-axis

y-z plane



cone $z=\sqrt{x^2+y^2}$

~~on y z pt.~~

on yz-plane $x=0$

$z=\sqrt{y^2}=y$

Slope is 1

bisects QI

so angle is $\frac{\pi}{4}$

volume : $\displaystyle\int_0^{2\pi}\int_0^{\pi/4}\int_1^{2} \underbrace{\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}_{dV} = \frac{14\pi}{3}\left(1 - \frac{\sqrt{2}}{2}\right)$

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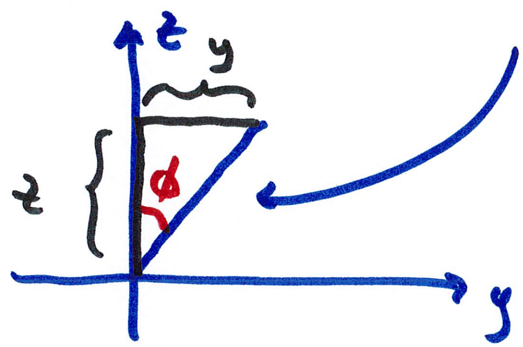
what if slope of cone is not 1 ?

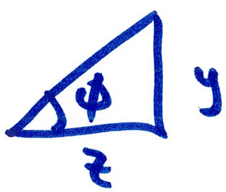
let's use $\sqrt{3}\, z = \sqrt{x^2+y^2}$ instead

on yz-plane plane:



$\sqrt{3}\, z = \sqrt{y^2} = y$

$\frac{z}{y} = \frac{1}{\sqrt{3}}$ $\qquad \frac{y}{z} = \sqrt{3}$



$\tan\phi = \frac{y}{z} = \sqrt{3}$

$\tan\phi = \sqrt{3}$

$\phi = \tan^{-1}(\sqrt{3})$

$\phi = \tan^{-1}(\sqrt{3}) = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \frac{\pi}{3}$

$0 \le \phi \le \frac{\pi}{3}$

review: mass (moments, center of ~~gr~~ mass)

average value of functions