The equation of the tangent plane to the graph of the function $f(x, y) = x - \frac{y^2}{2}$ at $(1, 2, -1)$ is

z

A. $2x + y + 4z = 0$

B. $x + 4y = 9$

C. $x - 2y - z = 2$

D. $-x + 2y + z = 2$

E. $x - y - 2z = 1$

define $F = x - \frac{1}{2}y^2 - z$

$\vec{\nabla}F = \langle 1, -y, -1 \rangle$

at $(1,2,-1)$: $\vec{\nabla}F = \langle 1, -2, -1 \rangle$

normal to surface

So use as normal of tangent plane

plane : $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$\langle a, b, c \rangle$: normal vector

(x_0, y_0, z_0) : point

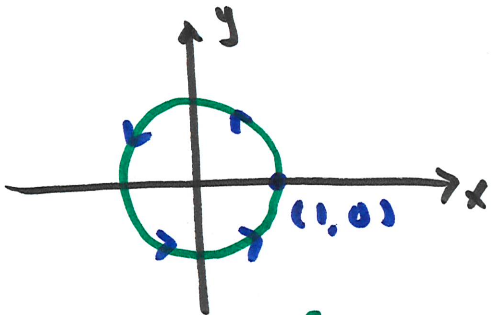
$(1)(x-1) - 2(y-2) - (z+1) = 0$

$x - 1 - 2y + 4 - z - 1 = 0$

$x - 2y - z = -2$

$-x + 2y + z = 2$

If C goes from $(1,0)$ counterclockwise once around the circle $x^2+y^2=1$, then $\int_C (x^2+2xy)dx+(x^2+2x+y)dy =$



A. 4π

B. -4π

C. -2π

D. 0

E. 2π

$$\vec{F} = \langle \underbrace{x^2+2xy}_{f},\ \underbrace{x^2+2x+y}_{g} \rangle \qquad d\vec{r} = \langle dx, dy \rangle$$

choices: parametrize C, then compute $\int_C \vec{F}\cdot d\vec{r}$

closed C, so can use Green's Theorem

Green's: $\int_C \vec{F}\cdot d\vec{r} = \iint_R (g_x - f_y)\, dA$

$g_x = 2x+2 \qquad f_y = 2x \qquad g_x - f_y = 2$

$$\int_C \vec{F}\cdot d\vec{r} = \iint_R 2\, dA = 2 \underbrace{\iint_R dA}_{\text{area of } R\text{: circle radius } 1} = 2\cdot\pi(1)^2 = 2\pi$$

Find the area of the parametric surface $\mathbf{r}(u,v) = u\cos v\mathbf{i} + u\sin v\mathbf{j} + v\mathbf{k}$, $0 \le u \le \pi$, $0 \le v \le u$.

A. $\frac{2}{3}(1+\pi^2)^{\frac{3}{2}}$

B. $\frac{1}{3}((1+\pi^2)^{\frac{3}{2}} - 1)$

C. $\frac{1}{2}(\sqrt{1+\pi^2} - 1)$

D. $\frac{1}{5}((1+\pi^2)^{\frac{3}{2}} - 1)$

E. $\frac{1}{5}(1+\pi^2)^{\frac{3}{2}}$

area: $\iint_R \overbrace{|\vec{r}_u \times \vec{r}_v| \, dA}^{dS}$

$\vec{r}_u = \langle \cos v, \sin v, 0 \rangle$

$\vec{r}_v = \langle -u\sin v, u\cos v, 1 \rangle$

$\vec{r}_u \times \vec{r}_v = \langle \sin v, -\cos v, u \rangle$

$|\vec{r}_u \times \vec{r}_v| = \sqrt{1+u^2}$ $\quad 0 \le u \le \pi, \quad 0 \le v \le u$

$$\int_{\substack{0\\u}}^{\pi} \int_{\substack{0\\v}}^{u} \sqrt{1+u^2} \, dv \, du = \int_0^{\pi} u\sqrt{1+u^2} \, du$$

Subs: $w = 1+u^2$
$dw = 2u\,du$

$$= \int_1^{1+\pi^2} \frac{1}{2} w^{1/2} \, dw = \frac{1}{3} w^{3/2} \Big|_1^{1+\pi^2}$$

$$= \frac{1}{3}\left[(1+\pi^2)^{3/2} - 1\right]$$

Let C be the curve $y = \frac{1}{3}x^3$, $0 \le x \le 1$. Compute $\int_C 12y\,ds$.

A. $8(2^{3/2} - 1)$

B. $12(2^{3/2} - 1)$

C. $\frac{2}{3}(2^{3/2} - 1)$

D. $\frac{3}{2}(2^{3/2} - 1)$

E. $2(2^{3/2} - 1)$

parametrize C: let $x = t$

then $\vec{r}(t) = \langle t, \frac{1}{3}t^3 \rangle$, $0 \le t \le 1$

then $ds = |\vec{r}\,'|\,dt$

$\vec{r}\,' = \langle 1, t^2 \rangle$ $\quad |\vec{r}\,'| = \sqrt{1+t^4}$

$\int_C 12y\,ds$

$\downarrow$

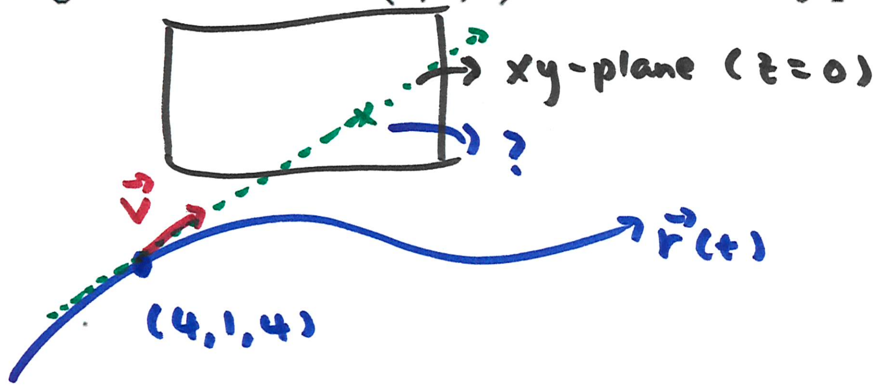
$|\vec{r}\,'|dt$

$= \int_0^1 12(\frac{1}{3}t^3)\sqrt{1+t^4}\,dt = \int_0^1 4t^3\sqrt{1+t^4}\,dt$ $\quad u = 1+t^4$, $du = 4t^3\,dt$

$= \int_1^2 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_1^2$

$= \frac{2}{3}\left[(2)^{3/2} - 1\right]$

Let C be the curve given by $\vec{r}(t) = \langle 4\sqrt{t}, t, 5 - t^2 \rangle$ for $t > 0$. At what point does the tangent line to C at $(4, 1, 4)$ intersect the xy plane?



A. $(0, 1, 0)$

B. $(4\sqrt{5}.\sqrt{5}, 0)$

C. $(2, 1, 0)$

D. $(8, 3, 0)$

E. $(0, -1, 0)$

line : $\vec{r}(t) = \vec{r}_0 + t\vec{v}$

$\vec{r}\,' = \langle \frac{2}{\sqrt{t}}, 1, -2t \rangle$ now need t so the point is $\langle 4, 1, 4 \rangle$

$\vec{r}(t) = \langle 4\sqrt{t}, t, 5 - t^2 \rangle$

we see $t = 1$

$\vec{r}\,'(1) = \langle 2, 1, -2 \rangle = \vec{v}$

tangent line : $\vec{\ell}(t) = \langle 4, 1, 4 \rangle + t \langle 2, 1, -2 \rangle$

$= \langle 4 + 2t, 1 + t, \underbrace{4 - 2t} \rangle$

$z = 0$ at intersection w/ xy-plane

$t = 2$

location at $t = 2$: $\vec{\ell}(2) = \langle 8, 3, 0 \rangle$

Find the maximum value of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$.

Lagrange is useable

A. 2

B. 4

C. 10

D. 16

E. 20

$f = x^2 + y^2$

$g = x^2 - 2x + y^2 - 4y = 0$

$\vec{\nabla} f = \lambda \vec{\nabla} g$ solve for x, y

$\langle 2x, 2y \rangle = \lambda \langle 2x-2, 2y-4 \rangle$

$$\left.\begin{array}{l} 2x = \lambda \cdot (2x-2) \rightarrow \lambda = \dfrac{2x}{2x-2} = \dfrac{x}{x-1} \\ 2y = \lambda \cdot (2y-4) \rightarrow \lambda = \dfrac{2y}{2y-4} = \dfrac{y}{y-2} \end{array}\right\} \quad \frac{x}{x-1} = \frac{y}{y-2}$$

$x(y-2) = y(x-1)$

$xy - 2x = xy - y \rightarrow y = 2x$

Sub into $x^2-2x+y^2-4y=0$

$$x^2-2x+(2x)^2-4(2x)=0$$

$$x^2-2x+4x^2-8x=0$$

$$5x^2-10x=0$$

$5x(x-2)=0 \rightarrow$ $x=0,$ $y=0,$ $\quad$ $x=2$ $y=4$

Points to check: $(0,0)$, $(2,4)$

compare $f=x^2+y^2$ $\rightarrow$ max at $(2,4)$, $f=20$

The triple integral $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{x^2+y^2}}(x^2+y^2)\,dzdydx$ when converted to cylindrical coordinates becomes:

A. $\int_0^{\pi}\int_0^{4}\int_0^{r} z^2 r\,dzdrd\theta$

B. $\int_0^{\pi}\int_0^{2}\int_0^{r} z^2 r\,dzdrd\theta$

C. $\int_0^{\pi}\int_0^{2}\int_0^{r} r^3\,dzdrd\theta$

D. $\int_0^{2\pi}\int_0^{2}\int_0^{r} r^3\,dzdrd\theta$

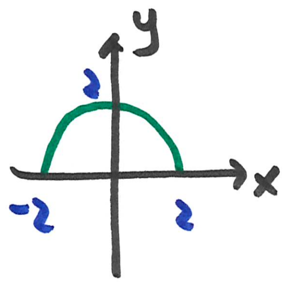
E. $\int_0^{2\pi}\int_0^{4}\int_0^{r} r^3\,dzdrd\theta$

$dz\,\underbrace{dy\,dx}$

"floor" xy-plane

$-2 \le x \le 2$

$0 \le y \le \underbrace{\sqrt{4-x^2}}_{\text{top of circle radius 2}}$



$0 \le r \le 2$

$0 \le \theta \le \pi$

$\underbrace{0}_{\text{xy-plane}} \le z \le \underbrace{\sqrt{x^2+y^2}}_{\text{cone}}$

in cylindrical, r

$\int_0^{\pi}\int_0^{2}\int_0^{r} \underbrace{r^2}_{x^2+y^2}\, \overbrace{r\,dz\,dr\,d\theta}^{dV}$

$= \int_0^{\pi}\int_0^{2}\int_0^{r} r^3\,dz\,dr\,d\theta$

Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the line segment from $P(1,0)$ to $Q(0,2)$ and $\mathbf{F}(x,y) = \langle ye^{xy}, xe^{xy} + \cos y \rangle$ is a **conservative** vector field. → curl $\vec{F} = \vec{0}$

A. $1 + \sin 2$

B. $\sin 2$

C. $1 - \sin 2$

D. $-1 + \sin 2$

E. $-1 - \sin 2$

Fundamental Theorem of Line Integrals:

if $\vec{F} = \vec{\nabla}\phi$ then

$$\int_C \vec{F} \cdot d\vec{r} = \phi(\text{end}) - \phi(\text{start})$$

find ϕ

$$\vec{F} = \langle ye^{xy}, xe^{xy} + \cos y \rangle = \vec{\nabla}\phi = \langle \phi_x, \phi_y \rangle$$

$$\phi_x = ye^{xy} \rightarrow \phi = \int ye^{xy}\,dx = e^{xy} + h(y)$$

$$\phi_y = xe^{xy} + \cos y = \phi_y = xe^{xy} + \frac{dh}{dy}$$

so, $\frac{dh}{dy} = \cos y \rightarrow h = \sin y + C$

$$\phi = e^{xy} + \sin y + C$$

$$\int_C \vec{F} \cdot d\vec{r} = \phi(\text{end}) - \phi(\text{start}) = \phi(0,2) - \phi(1,0) = (1 + \sin 2 + C) - (1 + C)$$

$$= \sin 2$$