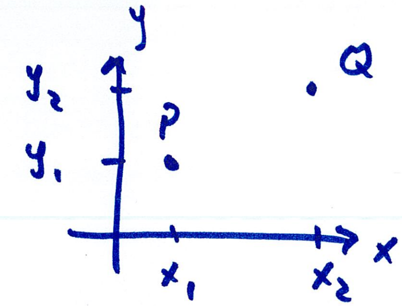
13.1 - 13.4 Review of Vectors

$P(x_1, y_1)$, $Q(x_2, y_2)$



vector from P to Q : $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$

$= (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$

destination x or y minus starting x or y

for example, $P(1, 2)$ $Q(3, -5)$

$\vec{PQ} = \langle 3-1, -5-2 \rangle = \langle 2, -7 \rangle$

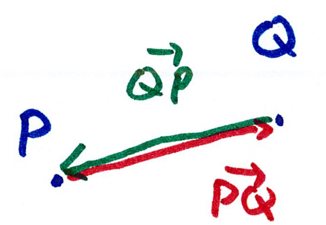
2 "steps" right

7 "steps" down

notice $\vec{QP} = -\vec{PQ}$

$\vec{QP} = \langle 1-3, 2 - -5 \rangle = \langle -2, 7 \rangle$

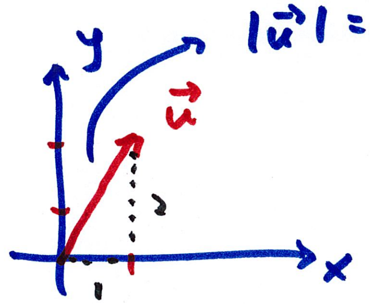
minus reverses direction



higher dimensions, same idea

magnitude / length of vector : $\vec{u} = \langle 1, 2 \rangle$

$|\vec{u}| = \sqrt{1^2+2^2}$



its length is $|\vec{u}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

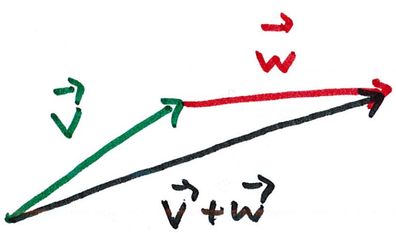
same idea in higher dimensions

addition / subtraction :

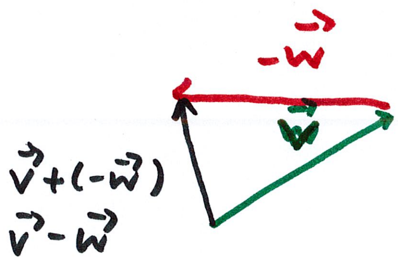
$\vec{v} = \langle 1, 2, 3 \rangle \quad \vec{w} = \langle 4, 5, 6 \rangle$

$\vec{v} + \vec{w} = \langle 1+4, 2+5, 3+6 \rangle = \langle 5, 7, 9 \rangle$

$\vec{v} - \vec{w} = \langle 1-4, 2-5, 3-6 \rangle = \langle -3, -3, -3 \rangle$



$\vec{v} - \vec{w} = \vec{v} + (-\vec{w})$



unit vector : a vector with magnitude of 1

$\vec{v} = \langle 1, 2, 3 \rangle$ is NOT a unit vector

because $|\vec{v}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} \neq 1$

a unit vector in same direction as $\vec{v}$:

$$\frac{\vec{v}}{|\vec{v}|} = \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

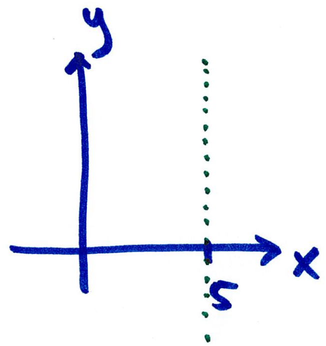
opposite direction as $\vec{v}$: $-\dfrac{\vec{v}}{|\vec{v}|}$

vector with length 3 in same direction as $\vec{v}$

$$3 \frac{\vec{v}}{|\vec{v}|}$$

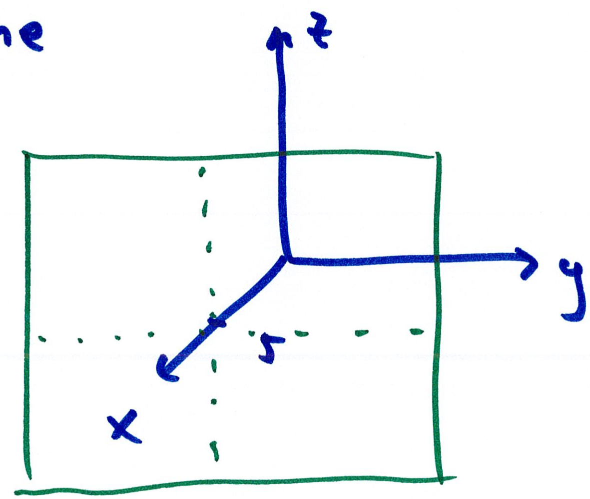
most shapes in 3D ($\mathbb{R}^3$) are very similar to their counterparts in 2D ($\mathbb{R}^2$)

for example, $x=5$ in $\mathbb{R}^2$ $\rightarrow$ all points with x of 5 and all possible y



in 3D, $x=5$ is a collection of all points w/ $x=5$, y, z all reals

this is a plane



Sphere : $(x-h)^2+(y-k)^2+(z-l)^2=r^2$

center : (h, k, l)
radius : r

vector dot product :

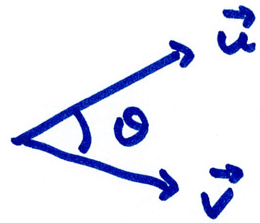
$\vec{u} = \langle 1, 2, 3 \rangle$

$\vec{v} = \langle 4, 5, 6 \rangle$

$\vec{u} \cdot \vec{v} = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 = 32$ a scalar

notice $\vec{v} \cdot \vec{u} = \vec{u} \cdot \vec{v}$

another formula: $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$



notice if $\vec{u} \cdot \vec{v} = 0$, then $\vec{u} \perp \vec{v}$

vector cross product

$\vec{u} = \langle 2, 1, 1 \rangle \qquad \vec{v} = \langle 5, 0, 1 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 5 & 0 & 1 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 1 \\ 5 & 0 \end{vmatrix}$$

$$= \vec{i}\,(1 \cdot 1 - 0 \cdot 1) - \vec{j}\,(2 - 5) + \vec{k}\,(0 - 5)$$

$$= \vec{i} + 3\vec{j} - 5\vec{k} = \langle 1, 3, -5 \rangle$$

$$\vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$$