13.6 Quadric Surfaces (part 1)

in $\mathbb{R}^2$ equations like $y = x^2$ are <u>curves</u>

in $\mathbb{R}^3$ equations in terms of x, y, z are <u>surfaces</u>

for example, $x^2 + y^2 + z^2 = 9$ is a sphere

$(x-2) + 3(y-1) + 5(z+4) = 0$ is a plane

normal vector: $\langle 1, 3, 5 \rangle$

point it goes thru: $(2, 1, -4)$

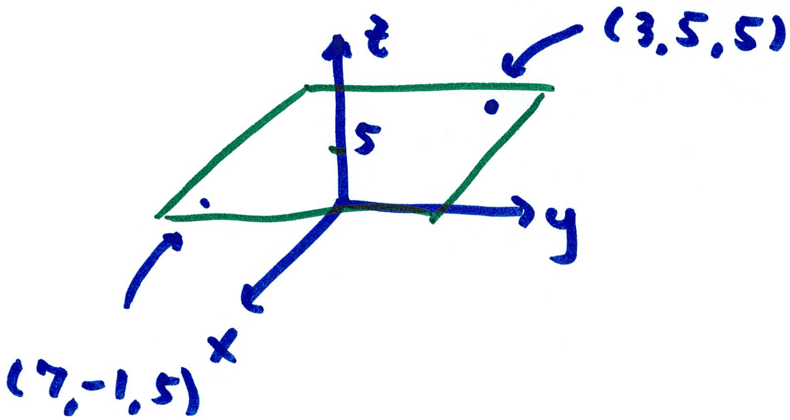
Sometimes a variable (or more) is missing (e.g. $x = 5$)

→ missing variable is "free"

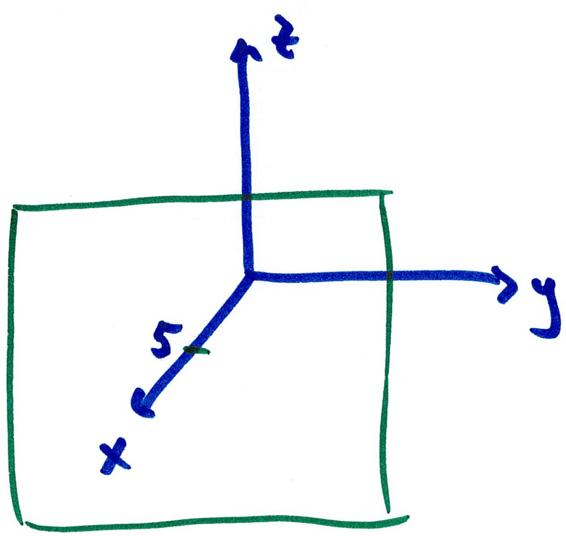
→ can take on all values in its domain

for example, $z = 5$ is missing x, y so x, y can be any real number

$\rightarrow$ collection of all points $(a, b, 5)$ $-\infty < a < \infty$, $-\infty < b < \infty$



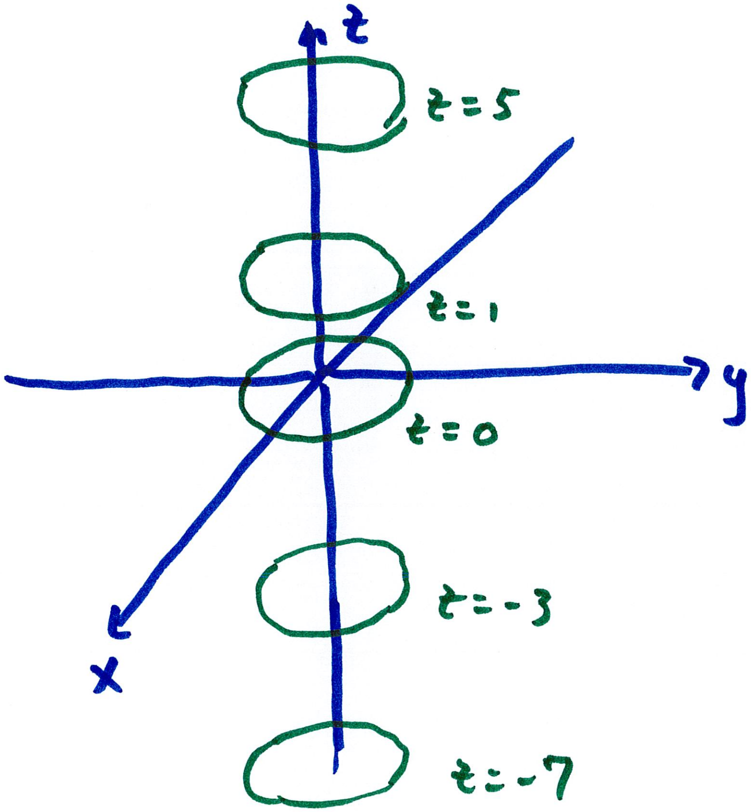
likewise, $x = 5$



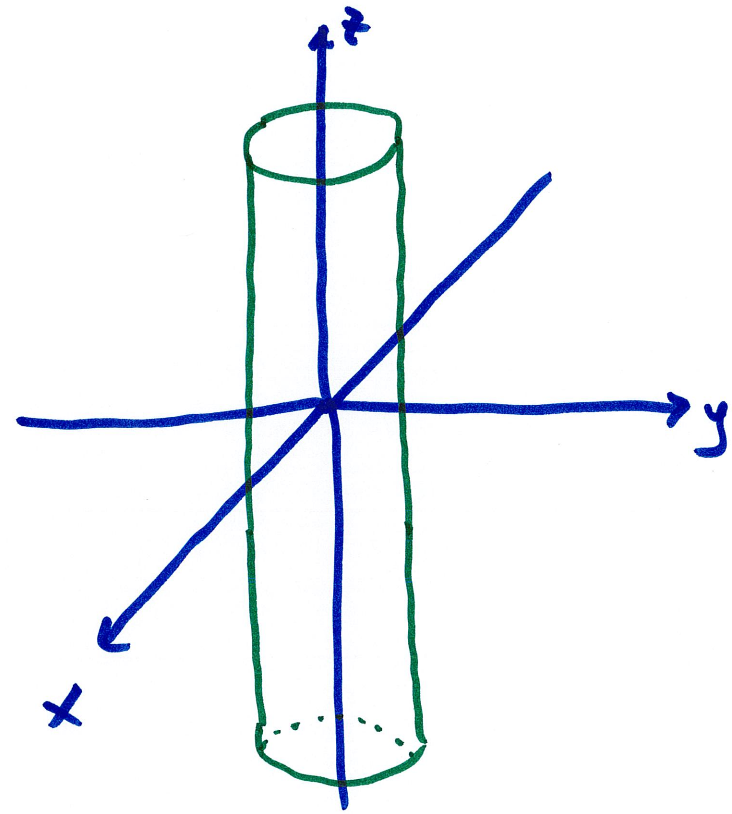
$x^2+y^2=1$ (in $\mathbb{R}^2$ is a circle)

z is missing, so $-\infty < z < \infty$

at any z, we have $x^2+y^2=1 \rightarrow$ circle radius 1

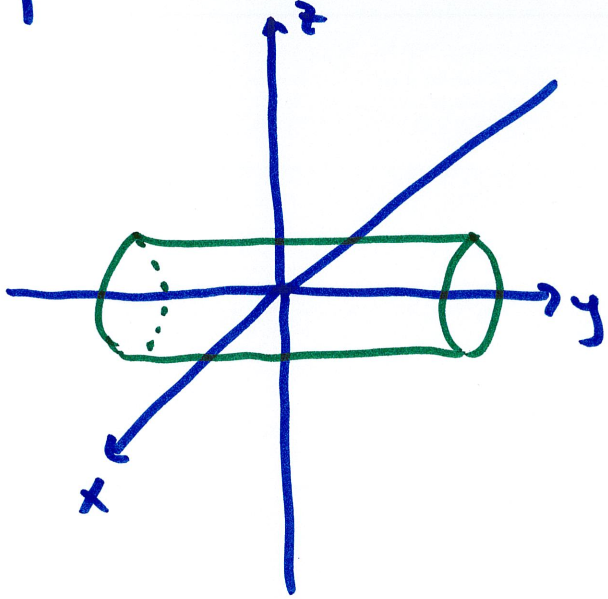


stack them $\rightarrow$

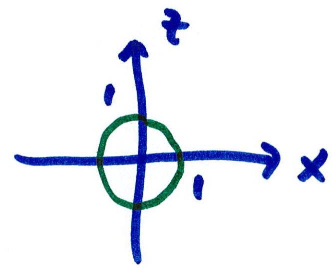


$x^2+z^2=1$?

$x^2 + z^2 = 1$



at any y



these slices we stack are called traces → intersections w/ a particular x, y, or z

intersection w/ xy-plane is called the xy-trace

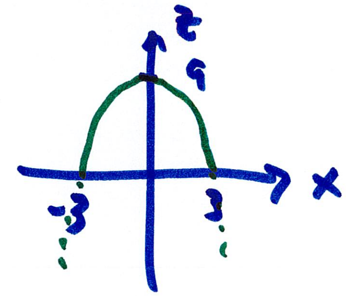
" " xz-plane " xz-trace

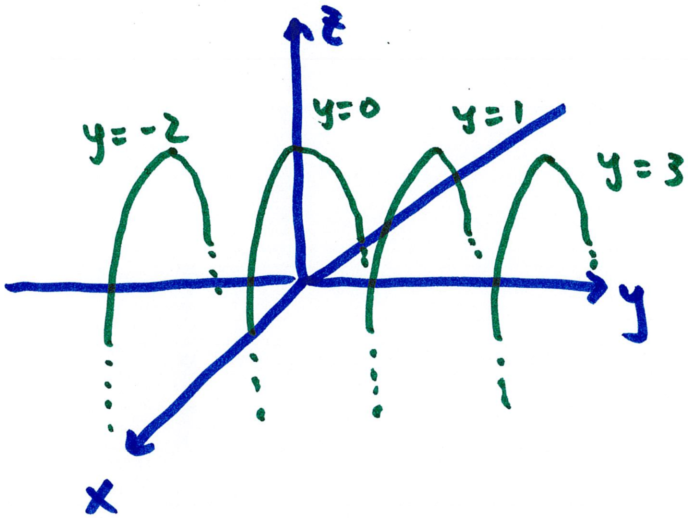
" " yz-plane " yz-trace

example $z = 9 - x^2$

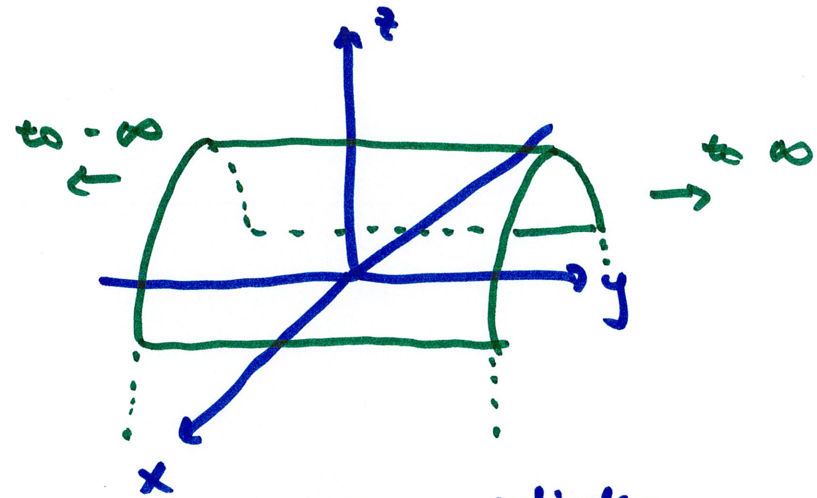
no y, so $-\infty < y < \infty$

at each y, we have a parabola





stack them



parabolic cylinder

example $x^2+y^2+z^2=16$ (pretend we didn't know this is a sphere)

let's examine its traces

xy-trace (intersection w/ xy-plane)

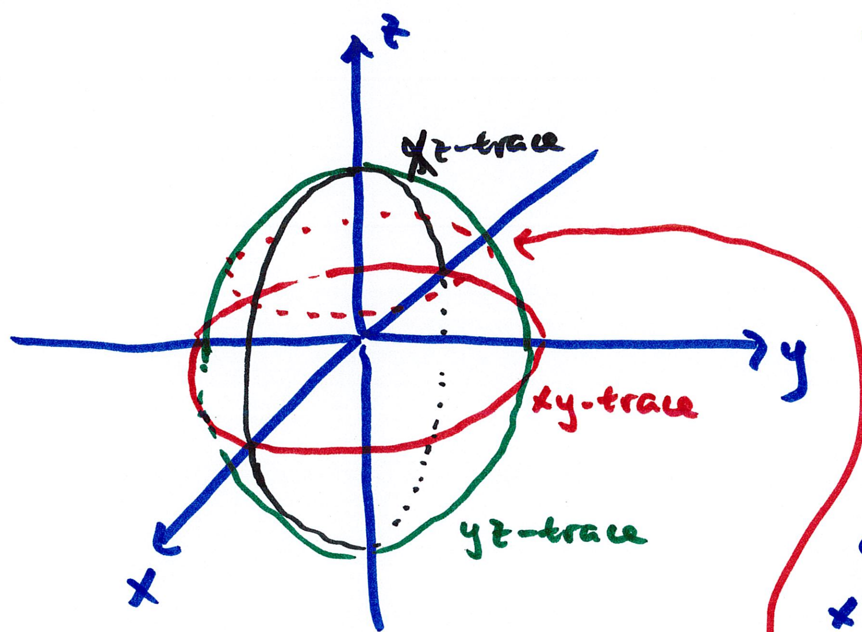
$z=0 \longrightarrow x^2+y^2=16$ circle radius 4

yz-trace

$x=0 \longrightarrow y^2+z^2=16$

xz-trace

$y=0 \longrightarrow x^2+z^2=16$



trace w/ $z=k$

$x^2+y^2=16-k^2$

circle radius $16-k^2$

here, $-4 \le k \le 4$

so, constraint on z is $[-4, 4]$

at $z=3$

$x^2+y^2=16-3^2=7$

circle radius $\sqrt{7}$

__example__ $x^2+y^2=z^2$

x-intercept : $x=0$

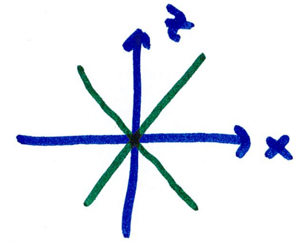
y-int : $y=0$

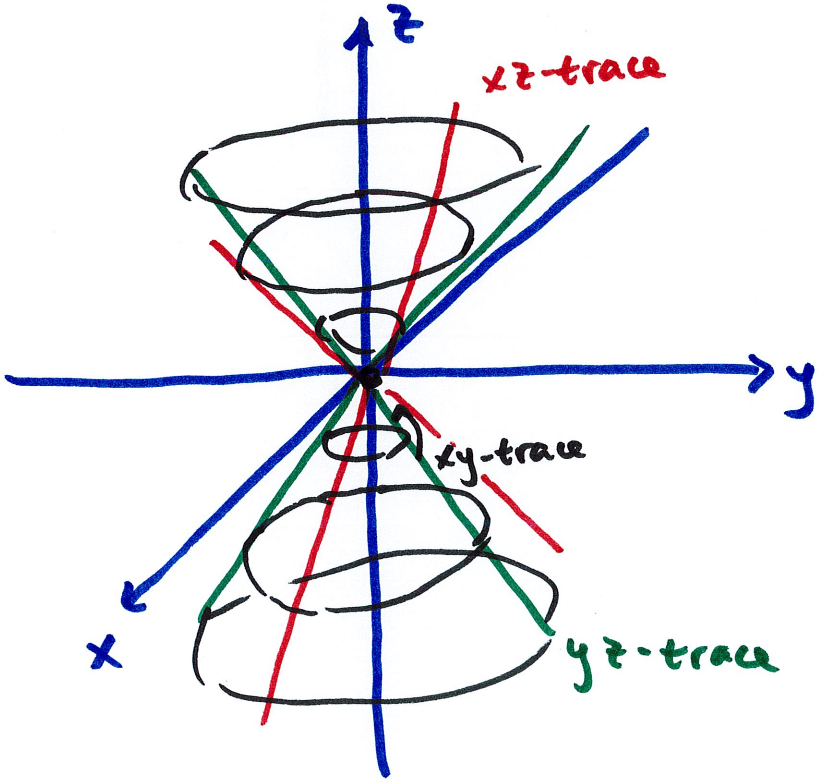
z-int : $z=0$

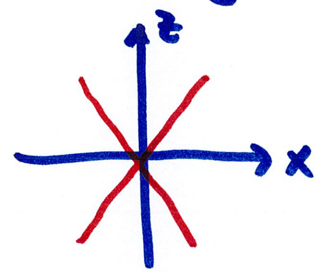
xy-trace : $(z=0)$ $x^2+y^2=0$ point at origin

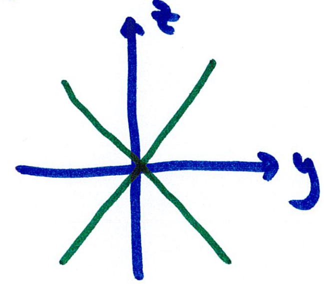
xz-trace : $(y=0)$ $x^2=z^2$ $z=\pm x$ lines

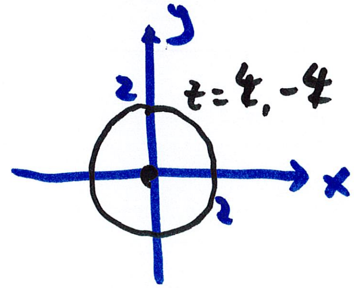
yz-trace : $(x=0)$ $y^2=z^2$ $z=\pm y$ lines







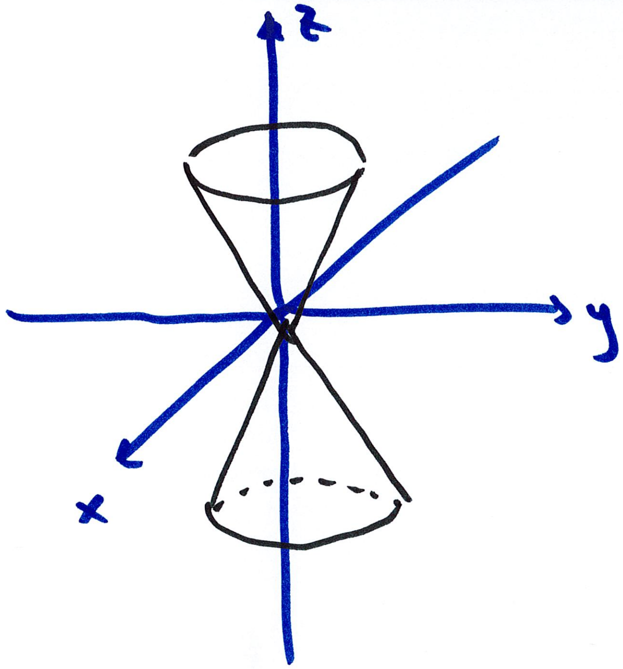




try trace w/ $z=k$

$x^2+y^2=k^2$

circle radius k



double cone

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example $z = x^2 + y^2$

xy-trace: $x^2+y^2=0$ point

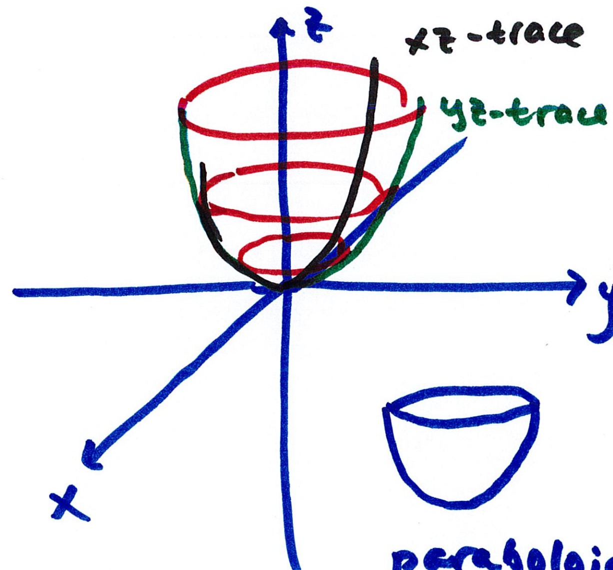
xz-trace: $z = x^2$ parabola

yz-trace: $z = y^2$ ״

slice at $z=k$

$x^2+y^2=k$ circle radius $\sqrt{k}$

note $z \geq 0$



paraboloid

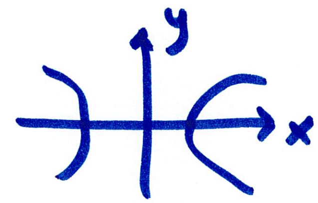
example $z = x^2 - y^2$

yz-trace: $z = -y^2$ parabola

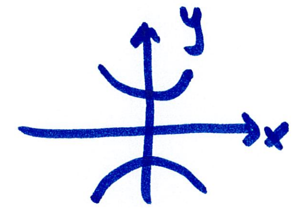
xz-trace: $z = x^2$ "

xy-trace: $x^2 = y^2$ lines

$z = k$ $x^2 - y^2 = k$ hyperbolas

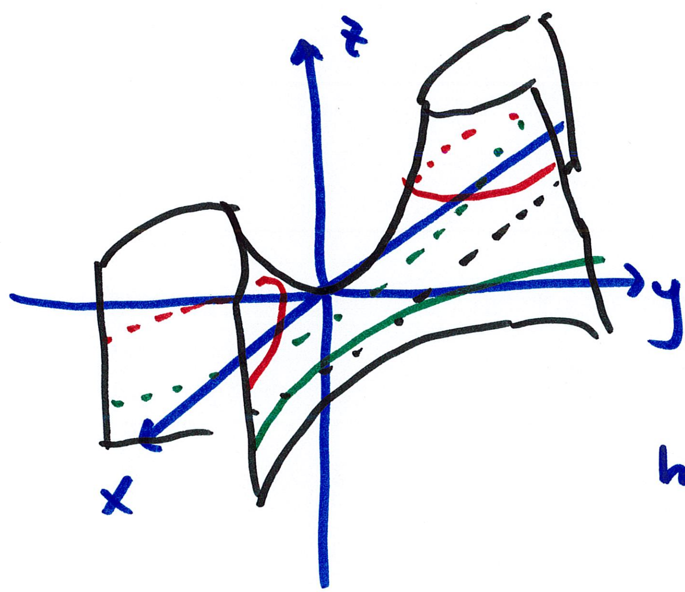


if $k > 0$



if $k < 0$

same for other perspectives



hyperbolic paraboloid