13.6 Quadric Surfaces (continued)

<u>example</u> $x^2+y^2-z^2=1$

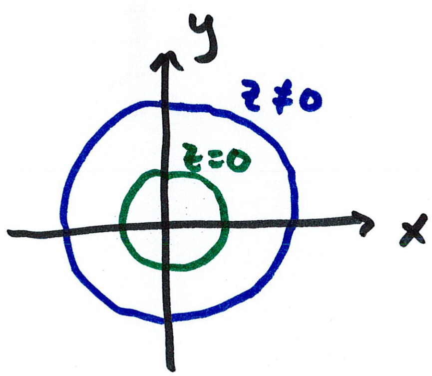
x-ints: $x^2=1 \to x=\pm 1$

y-ints: $y^2=1 \to y=\pm 1$

z-ints: $-z^2=1 \to$ no z-ints

xy-trace ($z=0$): $x^2+y^2=1$ circle radius 1

slices at other z: $x^2+y^2=1+z^2$ circles radius $\underbrace{\sqrt{1+z^2}}_{\geq 1}$



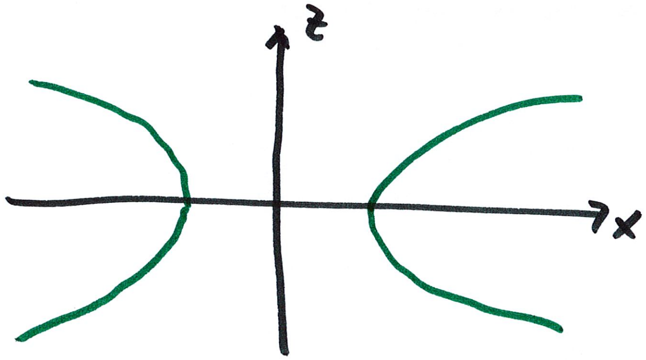
(larger circles than xy-trace)

$-\infty < z < \infty$

xz-trace ($y=0$): $x^2 - z^2 = 1$ hyperbola

intercepts: $x^2 = 1 \to x = \pm 1$

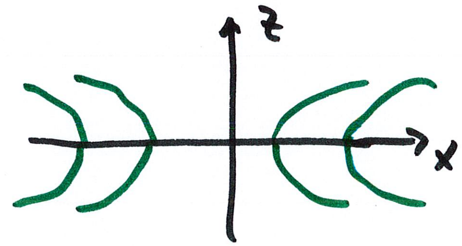
but no z-int



at other y: $x^2 - z^2 = 1 - y^2$

if ~~$|y^2|$~~ $|y| < 1$ $(-1 < y < 1)$ $\underbrace{1 - y^2 > 0}$

right side if pos. has x-int but no z-int

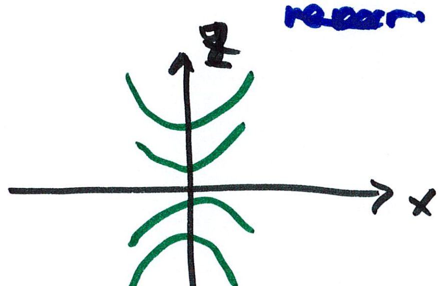


if $|y| > 1$ then $1 - y^2 < 0$

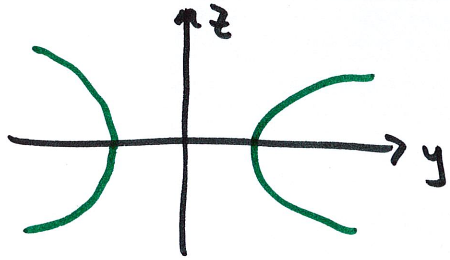
~~rearr~~ rearrang: $x^2 - z^2 = \overbrace{1 - y^2}^{<0}$

$z^2 - x^2 = \underbrace{y^2 - 1}_{>0}$

now has z-ints but no x-ints



yz-trace $(x=0)$ $\quad y^2-z^2=1$



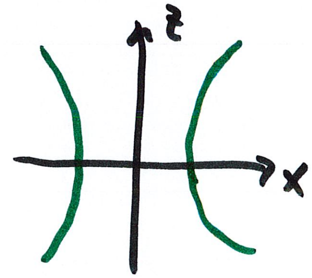
at other x: $\quad y^2-z^2=1-x^2$

switching hyperbolas depending on $|x|<1$ or not

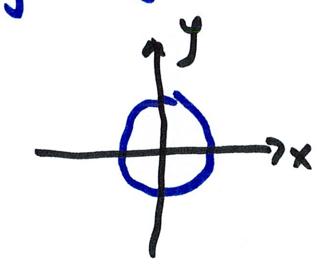
(like the other "side" perspective)

at $x=1$: $\quad y^2-z^2=0 \rightarrow$ lines (asymptotes of hyperbolas)

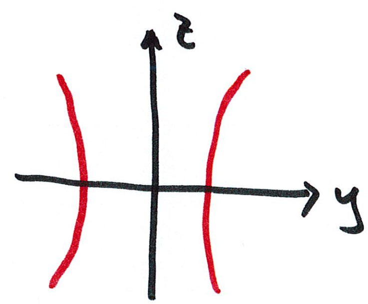
let's piece this together: $\quad$ xz-trace

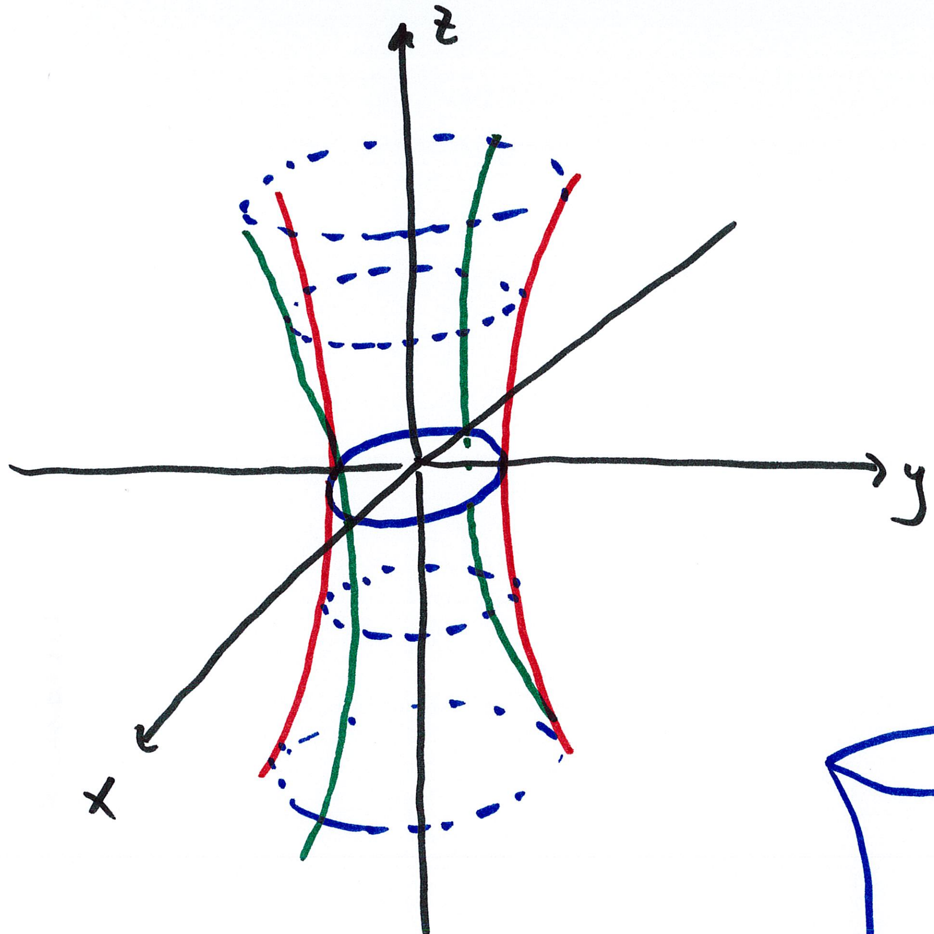


xy-trace

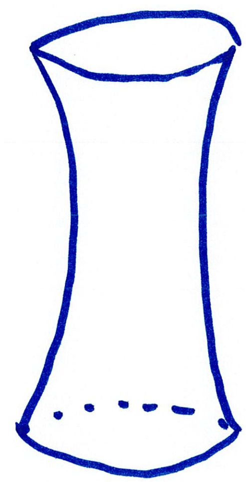


yz-trace





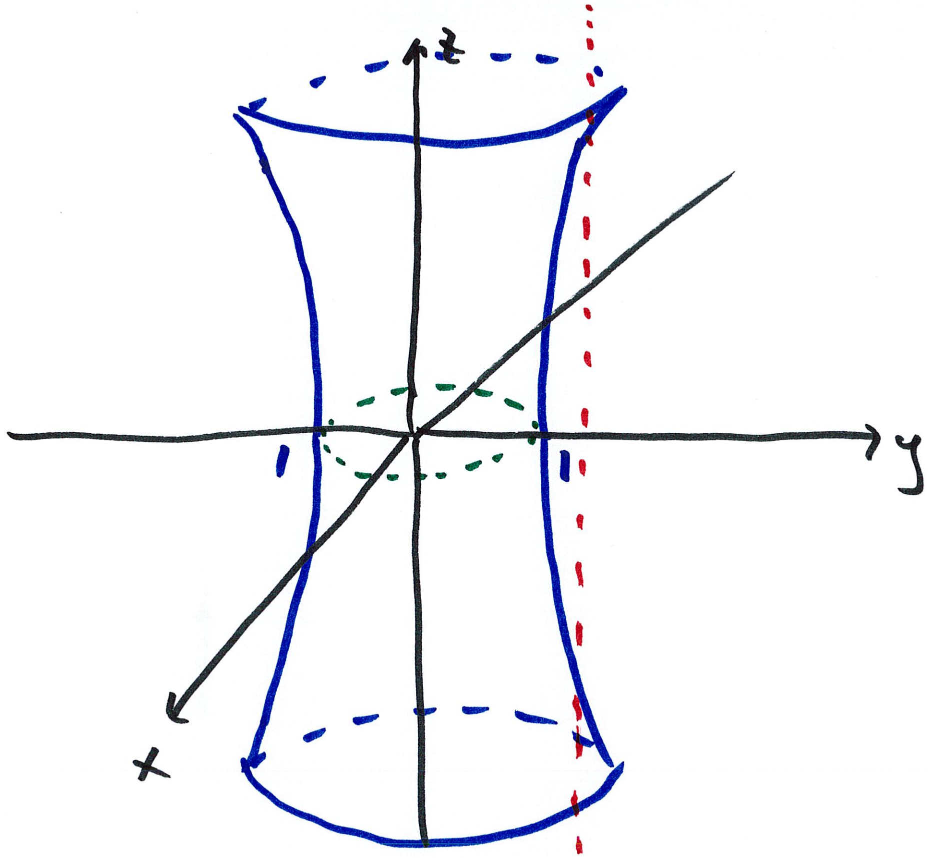
Circles get bigger as z changes



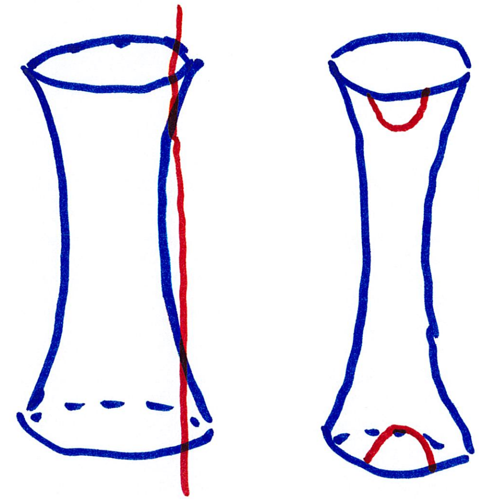
like a vase

"**Hyperboloid of one sheet**"

where are the switching hyperbolas?



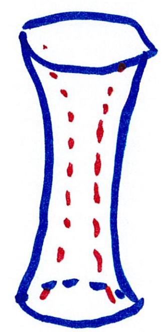
cut $y > 1$

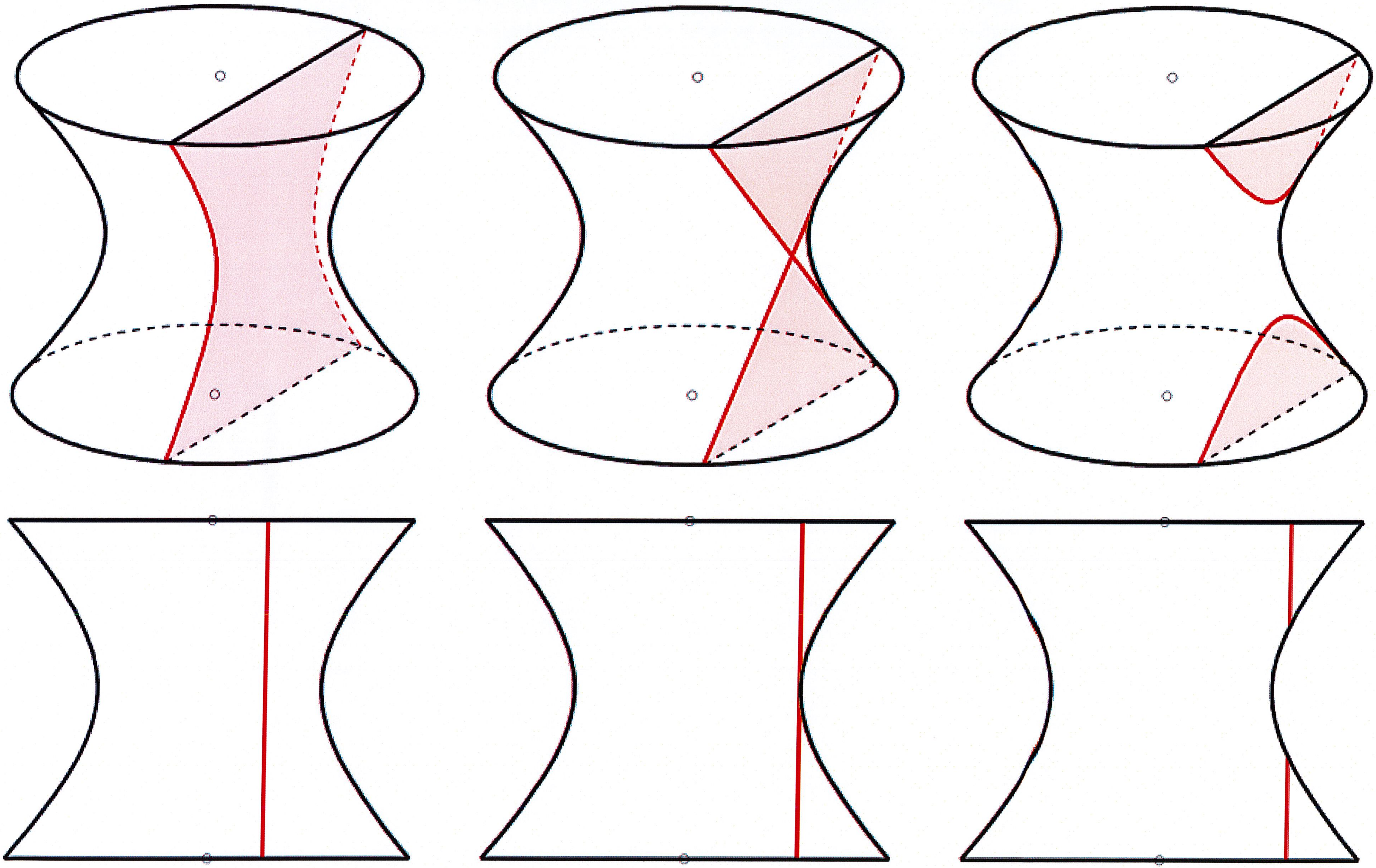


up/down hyperbolas

cut at $-1 < y < 1$

through the "neck"





example $-x^2-y^2+z^2=1$

x-ints: none

y-ints: none

z-ints: $z=\pm 1$

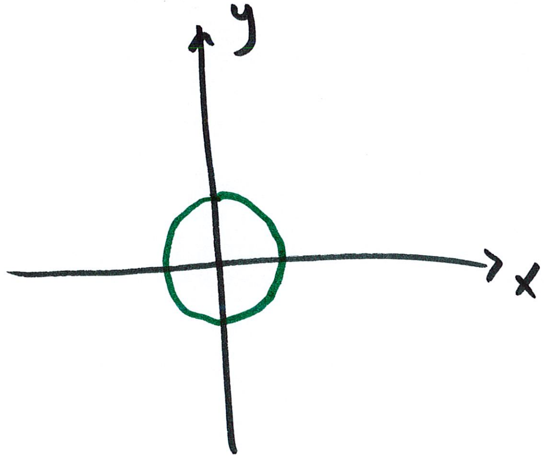
xy-trace ($z=0$): $-x^2-y^2=1$

$x^2+y^2=-1$ no such shape!

surface does not go through xy-plane

trace w/ other z: $-x^2-y^2=1-z^2$

$x^2+y^2=z^2-1$



circles radius $\sqrt{z^2-1}$

$|z|>1$

slices are circles

above $z=1$

below $z=-1$

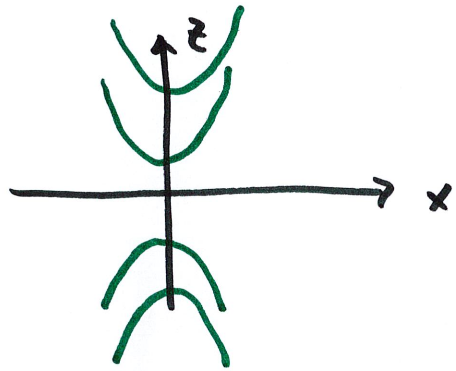
at $z=\pm 1$, points

xz-trace $(y=0)$: $-x^2+z^2=1$

$z^2-x^2=1$ hyperbolas w/ z-ints

at other y: $-x^2+z^2=1+y^2$

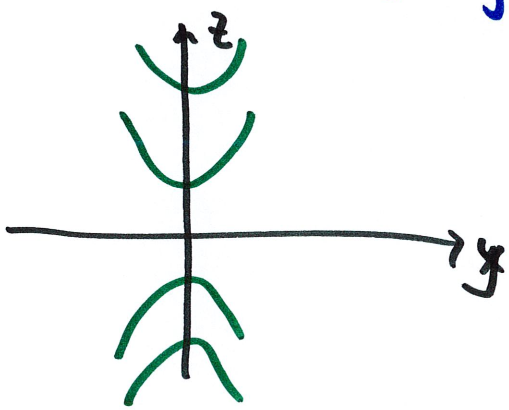
$z^2-x^2=1+y^2$ no matter what y is, always z-ints only → no switching

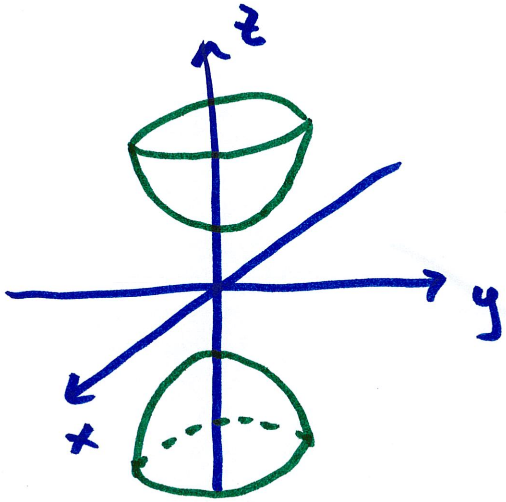


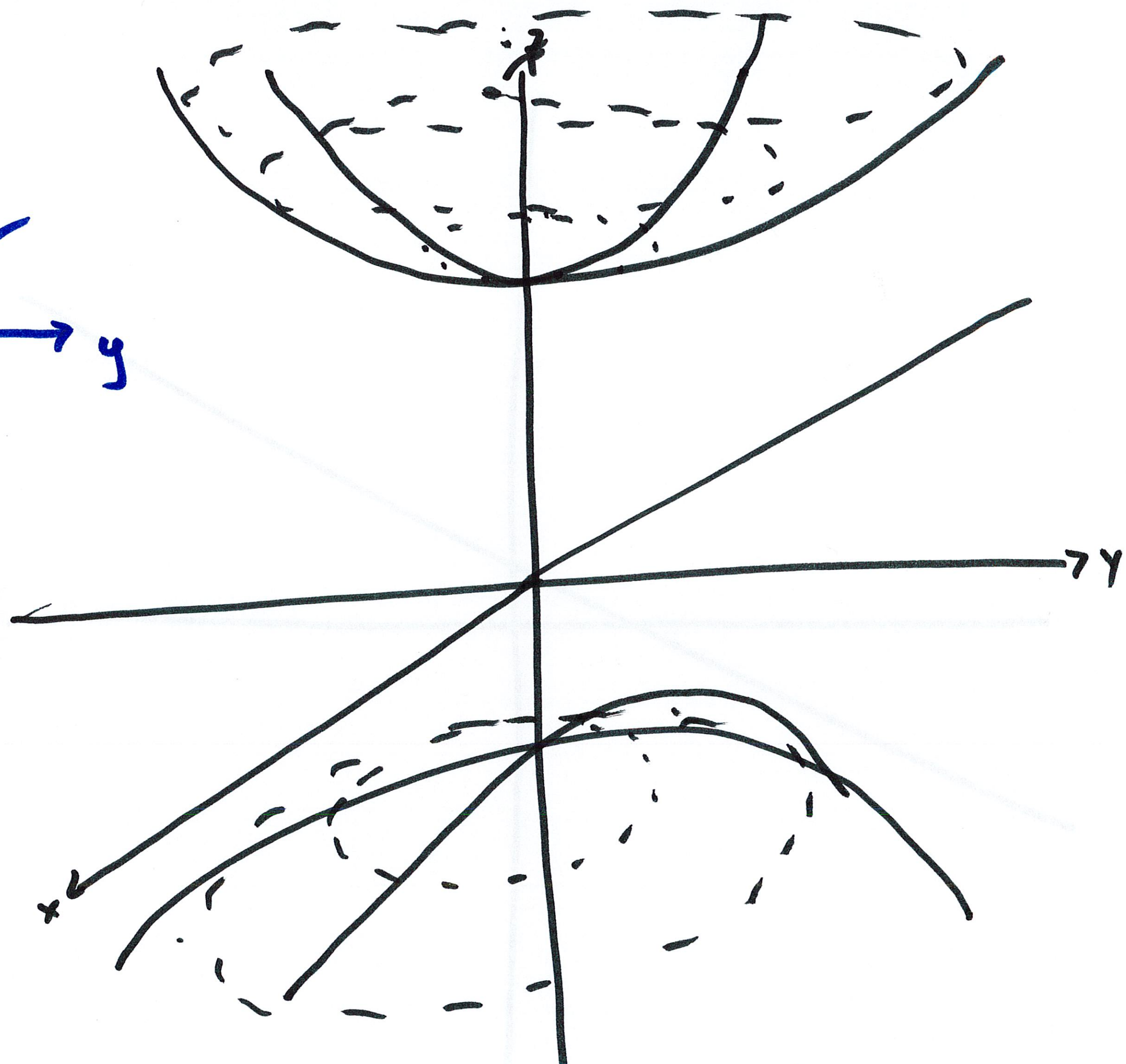
yz-trace $(x=0)$: $-y^2+z^2=1$

other x: $-y^2+z^2=1+x^2$

$z^2-y^2=1+x^2$ same idea, no switching





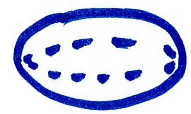


"Hyperboloid of two sheets"

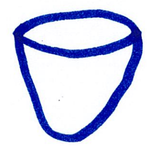
Quadric surfaces:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

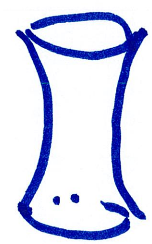
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ ellipsoids



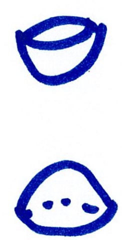
$$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$ paraboloids



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$ hyperboloid of one sheet

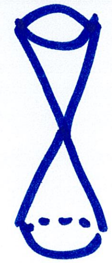


$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ " " two sheets



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

cone



$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

hyperbolic paraboloid