1.1 Intro to Differential Equations

differential equation (DE) : any equation containing **derivatives**

for example,

$$\frac{dy}{dx} = y(x)$$

$y(x)$: y function of x

y ← dependent

x ← independent variable

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2}$$

Newton's Gravitational Law

r

force $= \frac{GM}{r^2}$

planet mass M

$$\frac{dP}{dt} = K(P-L)$$

Logistic growth

P : population

L : population that can be sustained

order of DE: order of highest derivative

$y' = y$ first order

$r'' = -\frac{GM}{r^2}$ second order

goal: model physical ~~phenom~~ situations as DE's

solve DE's

understand solutions

what is a solution?

- function that satisfies the DE

for example, $y = e^x$ is a solution to $y' = y$

plug in

$y' = e^x$

$\underset{y'}{e^x} = e^x$ true, so $y = e^x$ satisfies $y' = y$

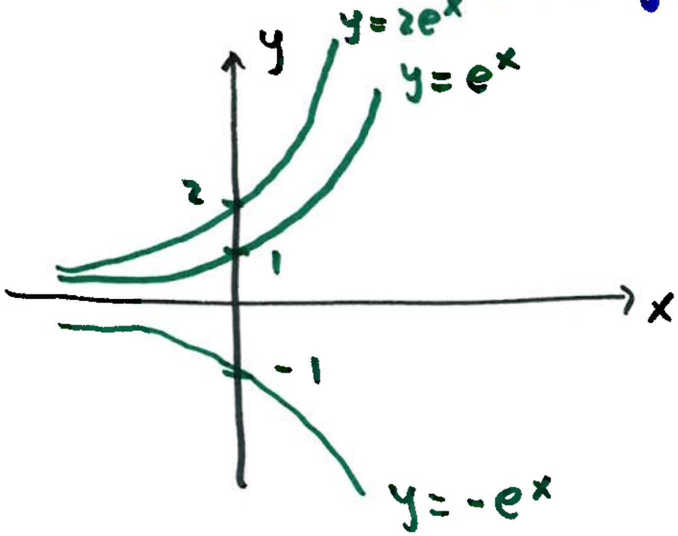
how about $y = 2e^x$? solution to $y' = y$?

$y' = 2e^x$ $\qquad$ $2e^x = 2e^x$ true

so $y = 2e^x$ is also solution

in fact, $y = Ce^x$ is solution to $y' = y$ for constant C

in DE's we often get a family of solutions



all are solutions to $y' = y$
but go through different points

$\underbrace{(0, C)}_{\text{"initial condition"}}$ $\qquad y = Ce^x$

example Find all solutions of $y''+y'-2y=0$ in the form of $y=e^{rx}$

(r is some constant)

$y=e^{rx}$ is a solution, so it satisfies the DE

$y'=re^{rx}$

$y''=r^2e^{rx}$

plug into DE: $r^2e^{rx}+re^{rx}-2e^{rx}=0$

$e^{rx}(r^2+r-2)=0$

$e^{rx}\neq 0$ for any x

so $r^2+r-2=0$

$(r+2)(r-1)=0$

$r=-2$ and $r=1$

So, solutions to $y''+y'-2y=0$ are $y=e^{-2x}$ and $y=e^{x}$

any constant-multiple is also a solution __example__ Find the constant C such that $y = \ln(x+C)$ is a solution to $e^y y' = 1$ and such that y goes through $(0,0)$.

first. verify $y = \ln(x+C)$ is a solution

need $y' \rightarrow \quad y' = \dfrac{1}{x+C}$

$\rightarrow x+C \qquad e^{\ln x} = x$

plug into $e^y y' = 1 \rightarrow \quad e^{\ln(x+C)} \cdot \dfrac{1}{x+C} = 1$

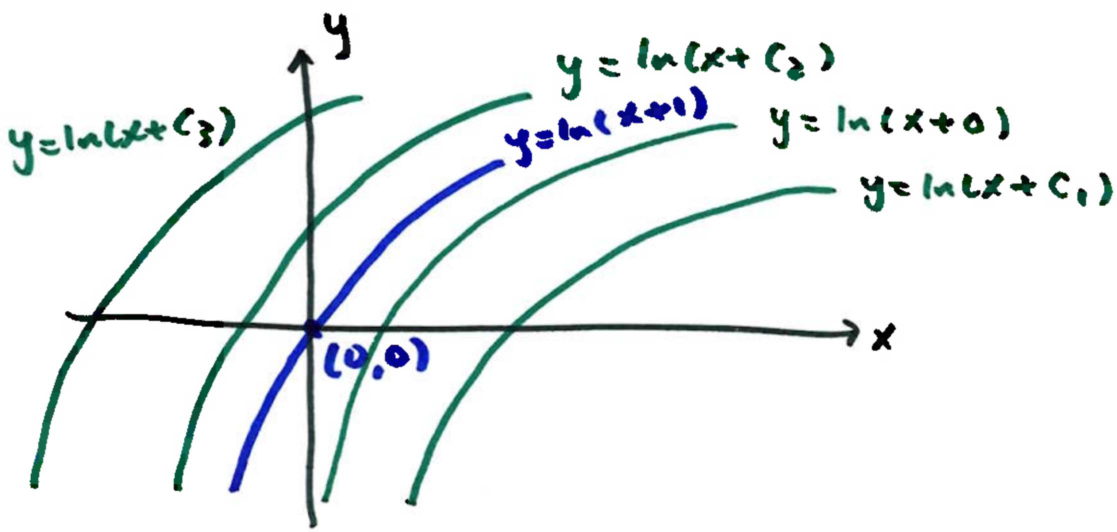
$x+C \cdot \dfrac{1}{x+C} = 1$ true

So, $y = \ln(x+C)$ is a solution

y goes through $(0,0)$

$0 = \ln(0+C) = \ln(C) \rightarrow \quad C = 1$

So, $y = \ln(x+1)$ is the only solution


y
y = ln(x+ C₂)
y=ln(x+C₃)
y=ln(x+1)
y = ln(x+0)
y=ln(x+C₁)
x
(0,0)