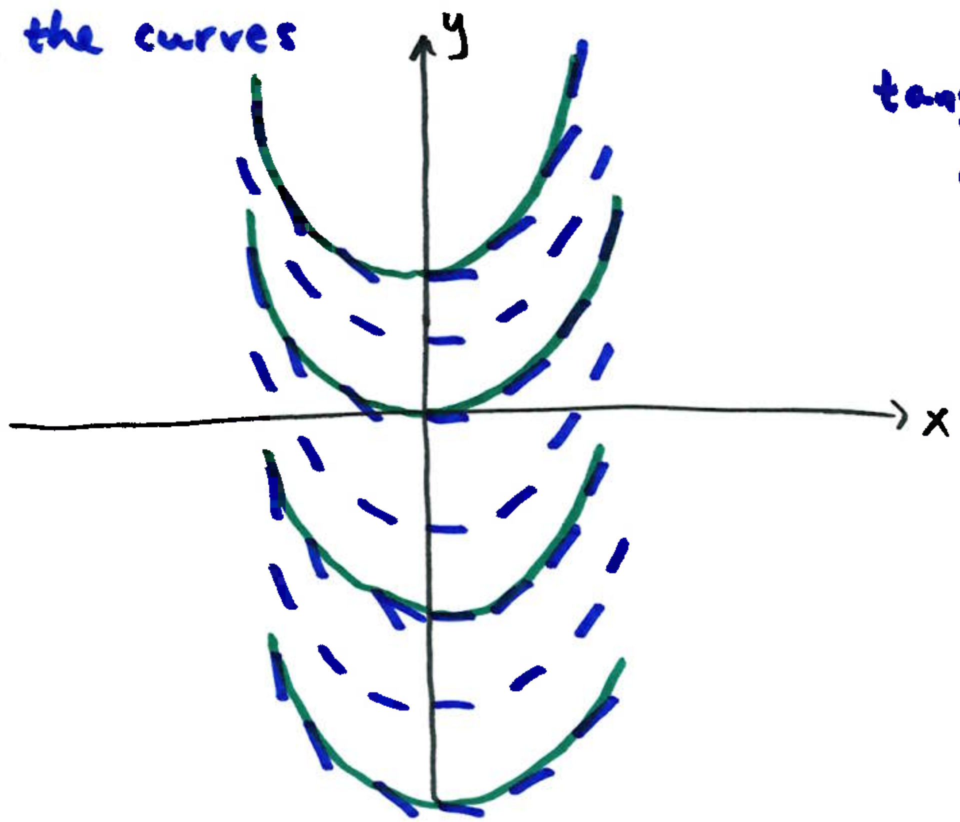
1.3 Slope Field and Solution Curves

Qualitative view of solutions

example from last time : $\frac{dy}{dx} = 2x$

$$y = \int 2x\,dx = x^2 + C$$

sketch the curves

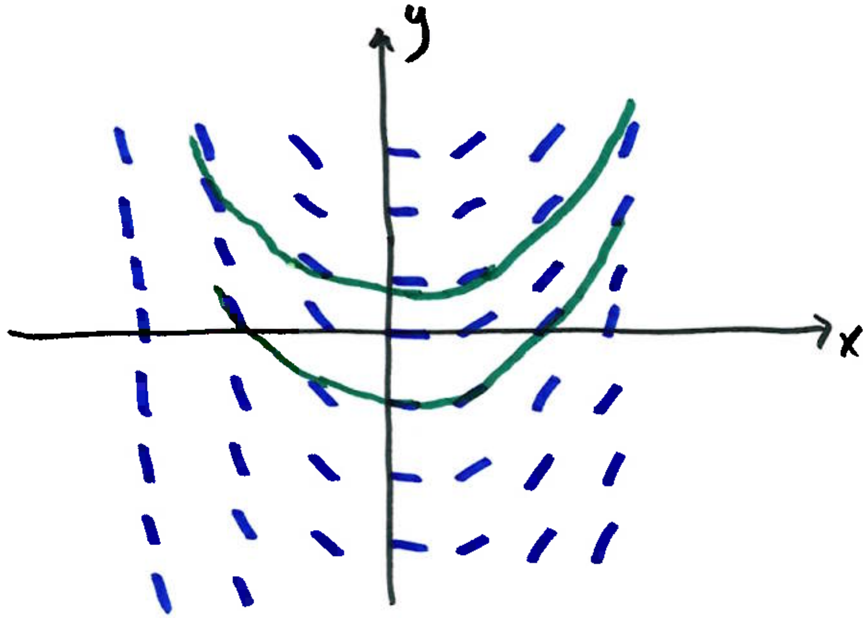


tangent lines on the curves

slope of each is $\frac{dy}{dx} = 2x$

(DE or differentiate solution)

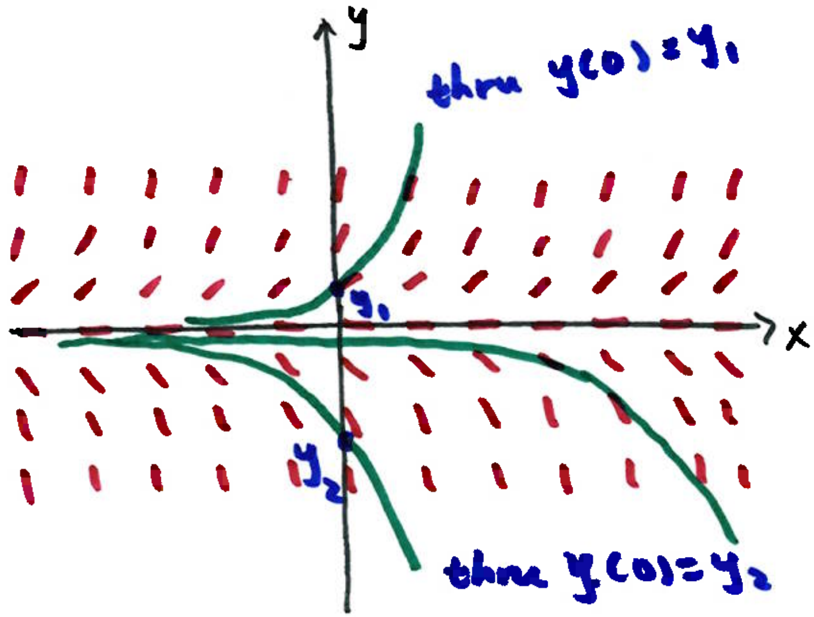
remove the parabolas (solution curves)



each has slope $\frac{dy}{dx} = 2x$

we can "see" solution curves (whose tangent lines have slopes $2x$) hidden in the slope field

example $\frac{dy}{dx} = y$



slope is $\frac{dy}{dx} = y$

no dependency on x, moving left or right the slopes stay the same

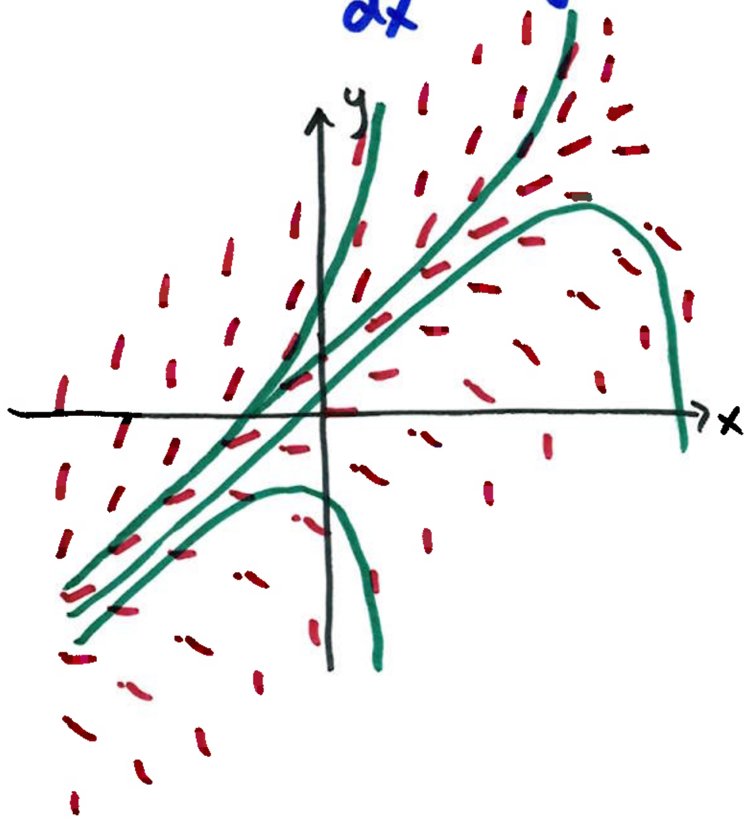
where are slopes zero?

$y' = y = 0 \rightarrow$ along curve $y = 0$

above $y = 0 \rightarrow y > 0 \rightarrow y' = y > 0$

positive slopes $\rightarrow$ high steeper lines guess solution : exponential , maybe $y = e^x$ or $y = -e^x$

<u>example</u> $\frac{dy}{dx} = y - x$



slopes depend on x and y

start with where $y' = 0$

$y' = y - x = 0 \rightarrow$ on curve $y = x$

above it: $y > x \rightarrow y - x > 0$

so $y' > 0$

positive slopes above

going up $\rightarrow$ more positive

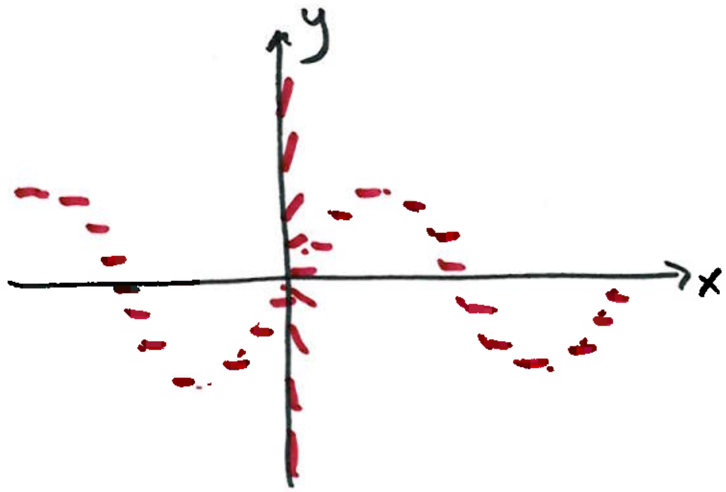
Same idea below, but negative slopes

notice if $y(0) < 0$, as $x \to \infty$, $y \to -\infty$

if $y(0) > 0$, as $x \to \infty$, $y \to \infty$

~~if y~~(

<u>example</u> $y' = y - \sin x$



$y' = 0$ on curve $y = \sin x$

above $y = \sin x$ $y' > 0$

y inc $\rightarrow$ y' inc

below $y = \sin x$ $y' < 0$

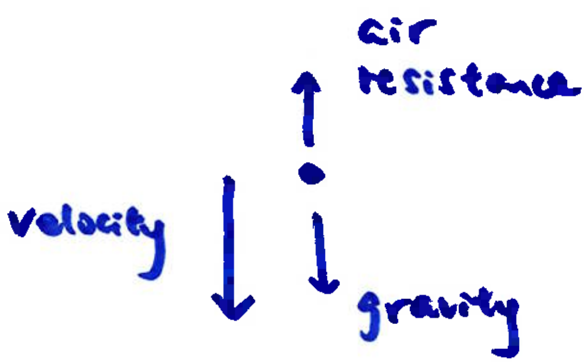
y dec $\rightarrow$ y' dec

fix y, move x right ?

fix y, move x left ?

application: terminal velocity

free fall, acceleration due to gravity = acceleration due to air resistance



DE: Newton's Law $F = ma$

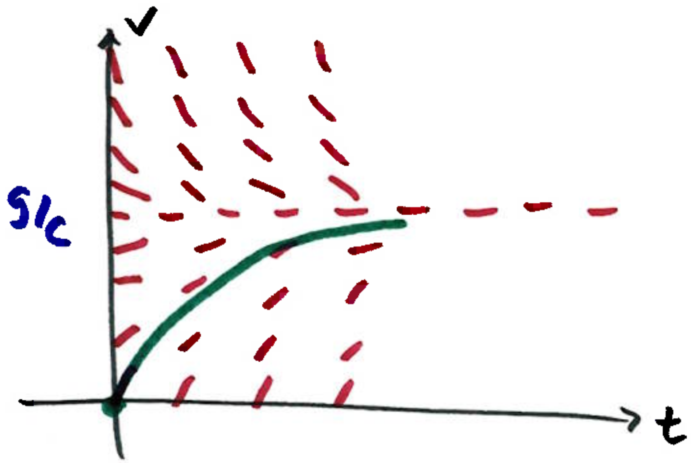
$$m\frac{dv}{dt} = \underbrace{mg}_{\text{gravity}} - \underbrace{D}_{\text{drag}}$$

$$\frac{dv}{dt} = g - a_D$$ acc due to drag

popular models of a_D: cv, cv^2 (c constant)

$$\frac{dv}{dt} = g - cv$$

slope field: slope $= 0$ along $v = \frac{g}{c}$



below $v = \frac{g}{c}$, $v' > 0$

above $v = g/c$, $v' < 0$

terminal: $v = g/c$

high c → low terminal → survive

(parachute)