7.4 Solution Curves of Linear Systems (part 2)

Last time: eigenvectors form straight line solutions
visible on phase portraits

positive λ : away from origin (source)

negative λ : toward origin (sink)

nodes: improper (common)

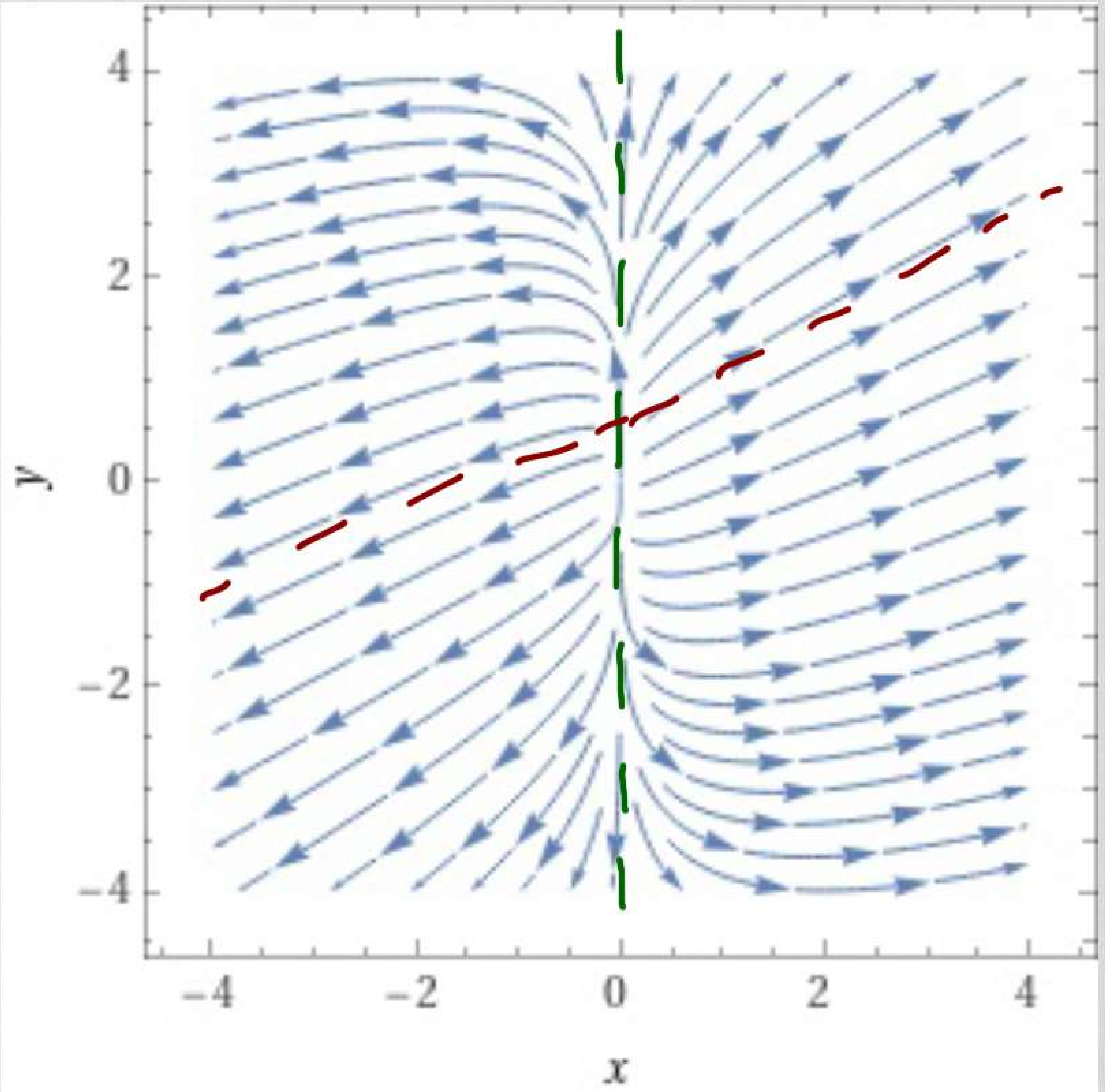
→ solutions leave or approach
origin on asymptote

→ distinct λ's, repeated λ's w/o
enough eigenvectors

proper

→ repeated λ's w/ enough eigenvectors

example



improper nodal source

all solutions flow away from origin → BOTH λ's > 0

source

improper because all solutions follow an asymptote leaving the origin

eigenvectors : straight lines?

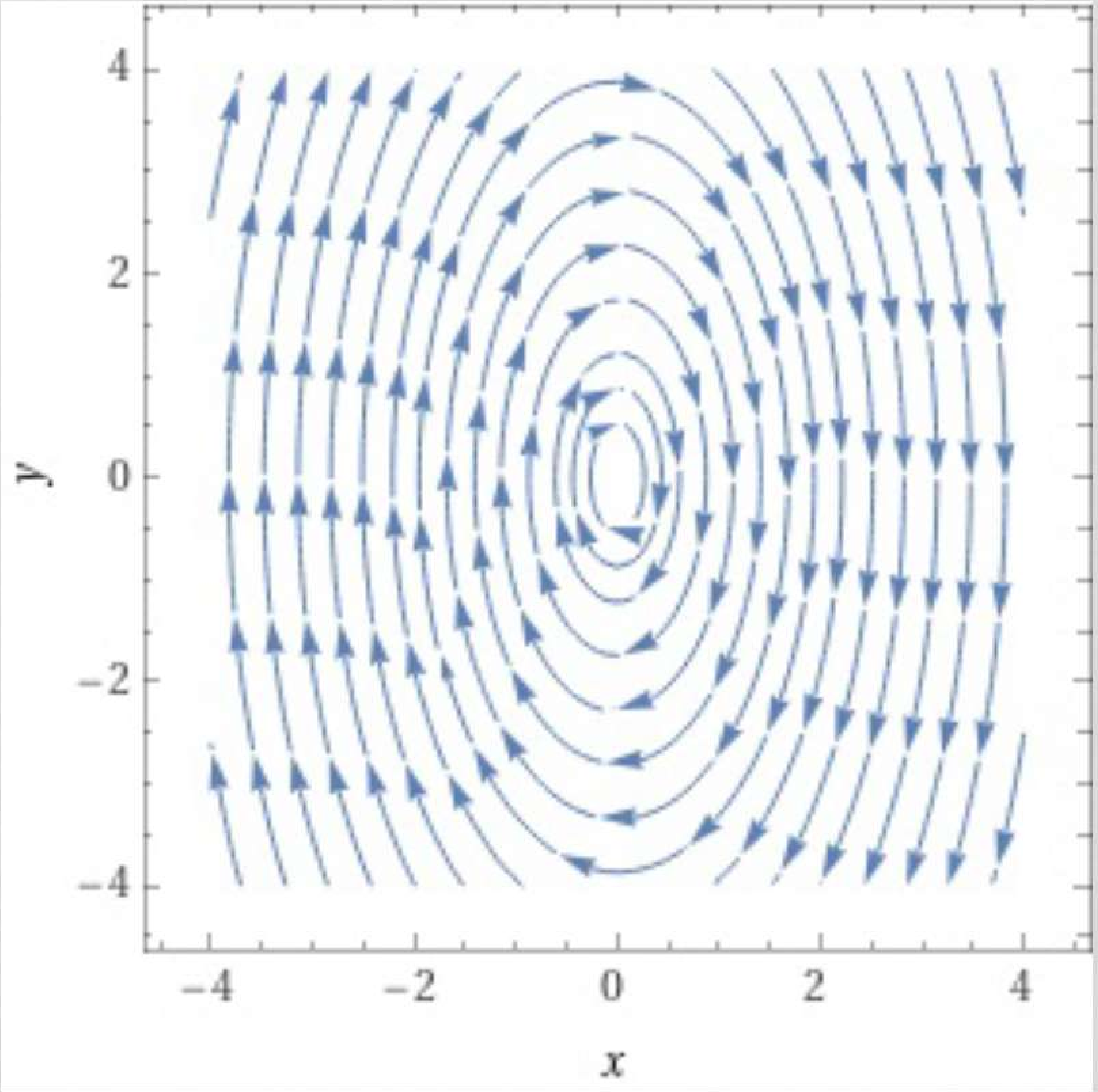
one is the green dashed line

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

the other is the red one

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

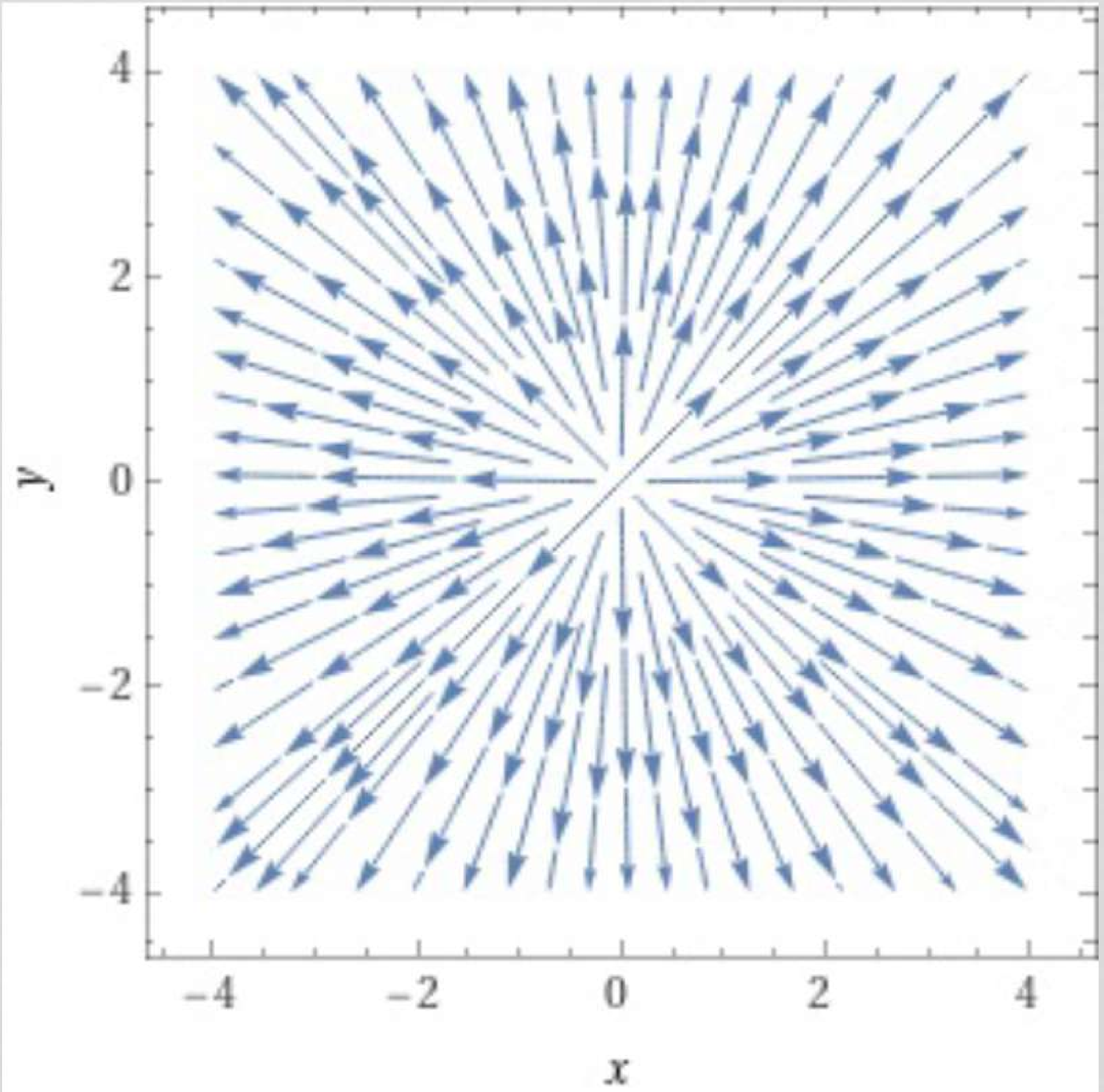
example



center

ovals/spirals → complex λ's
(because solutions are sine and cosine → periodic)
Not going into or away from origin → purely imaginary λ
(into origin → real part of λ is < 0, away → real part of λ is > 0)
Complex λ's → generally cannot see enough to determine eigenvectors

example



proper nodal source

proper: not following asymptotes

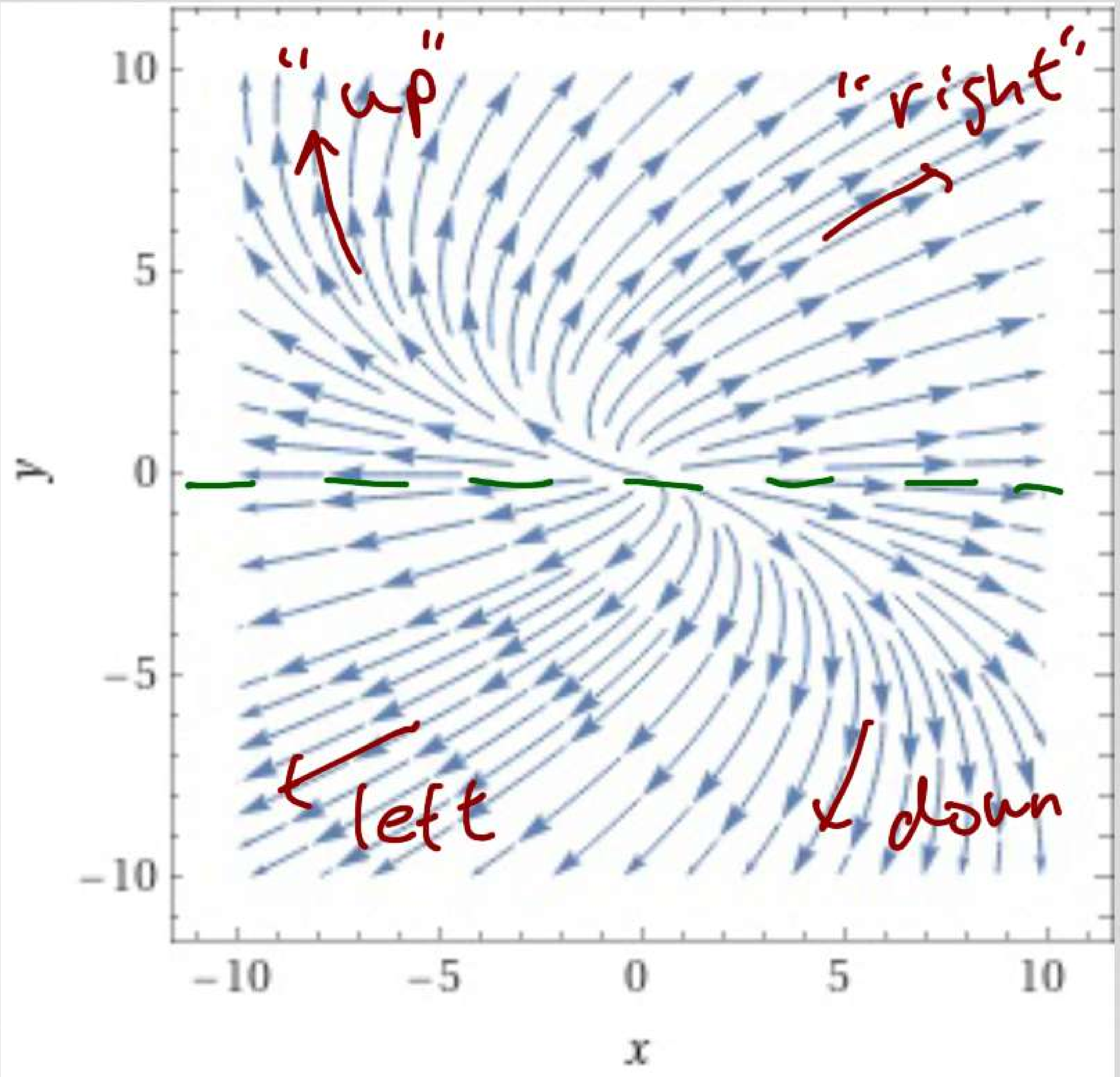
↳ repeated λ's with enough eigenvectors (not generalized)

λ's: positive (source)

eigenvectors: pick ANY two linearly independent vectors

pick $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

example



improper nodal source

$\lambda > 0$ for both
(solutions flow away)

λ's are either repeated
w/o enough eigenvectors

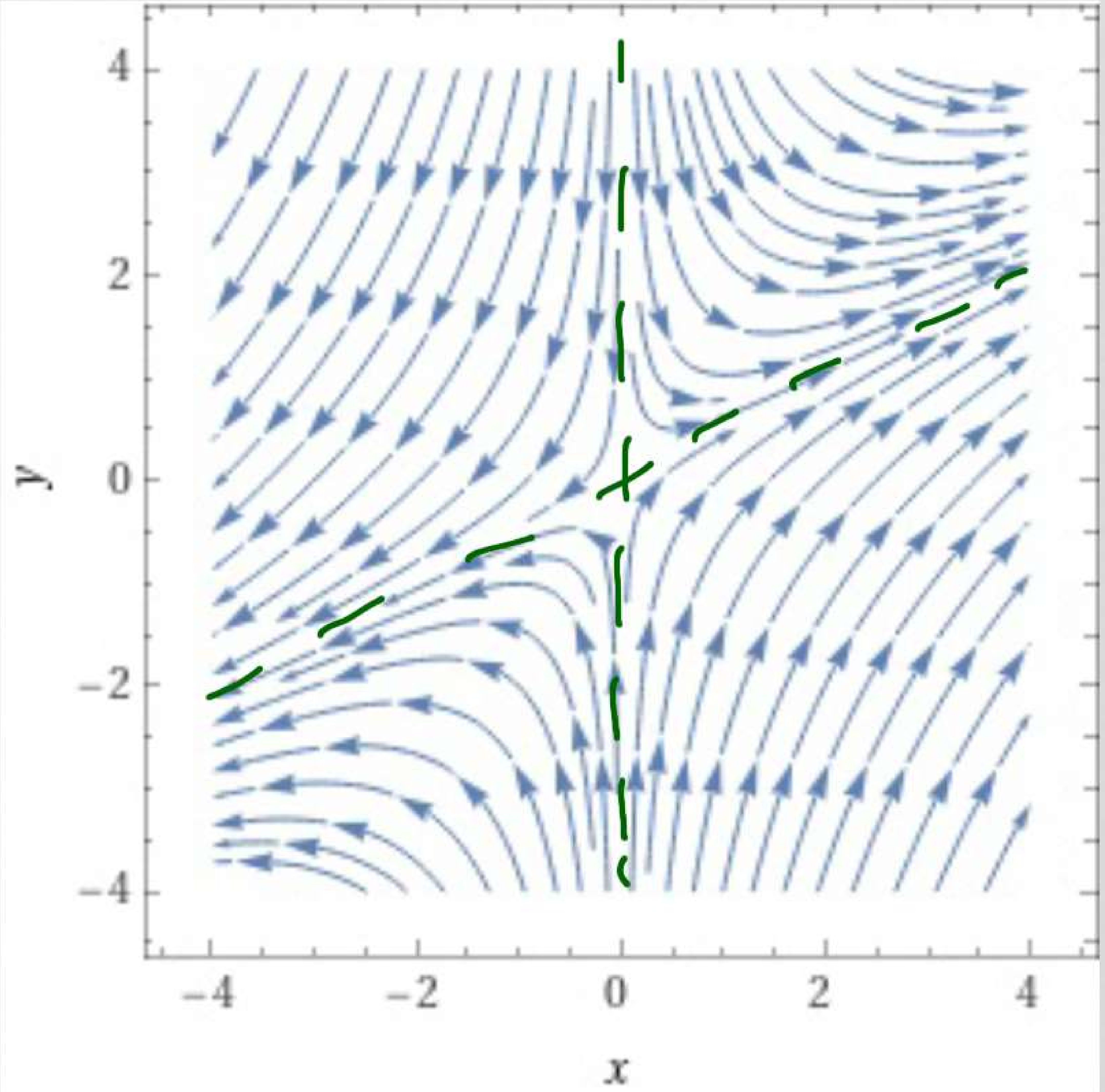
OR λ's are distinct

to tell, try to spot straight
line solutions

one is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

but no other one visible
→ most likely a repeated
pair of λ's w/o enough
eigenvectors

example



saddle point

saddle pt: λ's opposite signs

eigenvectors: asymptotes

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ w/ $\lambda < 0$ (toward origin)

$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ w/ $\lambda > 0$ (away)