$y' + p(x)\, y = Q(x)$

integrating factor is $I = e^{\int p(x)\,dx}$

why?

goal: find I such that $Iy' + pIy = \frac{d}{dx}(Iy)$

expanding the right side we get

$Iy' + pIy = Iy' + I'y$

so we see $pIy = I'y$

or $pI = \frac{dI}{dx}$ (this is separable!)

$$\frac{1}{I}\,dI = p\,dx$$

$$\ln I = \int p\,dx$$

$$I = e^{\int p\,dx}$$