3.1 Introduction to Linear Systems

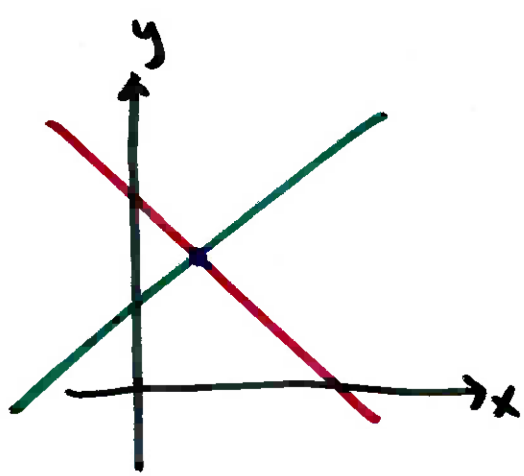
two equations of two variables

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

to "solve" the system → find (x, y) that satisfies BOTH equations

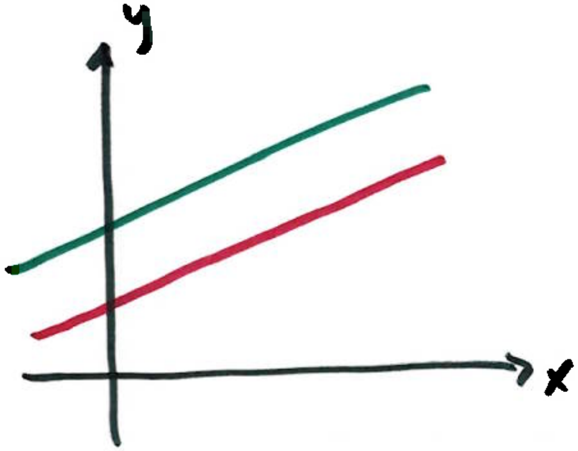
geometric view: each is a line

(x, y) working in BOTH eqs is equivalent to finding point(s) that is/are on BOTH lines.



intersect once → only one (x, y) on both lines

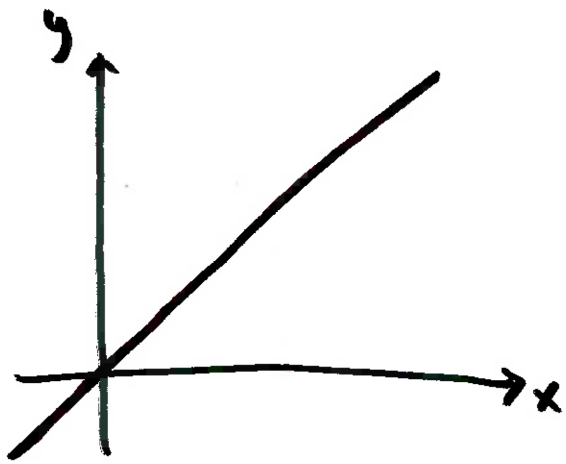
<u>one</u> solution



parallel, never intersect

no points on BOTH lines

no solution



lines on top of each other

every point on one line is a point on the other line

infinitely-many solutions

example

$$x + 3y = 9 \quad - \text{①}$$
$$2x + y = 8 \quad - \text{②}$$

one method – method of elimination

add multiple of one equation to the other such that one variable vanishes if we choose to eliminate y, we can multiply ② by -3

~~and~~ and add to ①

$-3② + ① \longrightarrow$

$$\left.\begin{aligned} -6x - 3y &= -24 \\ x + 3y &= 9 \end{aligned}\right\} \text{add}$$

$$-5x = -15$$

$\boxed{x = 3}$ go to any equation to find y

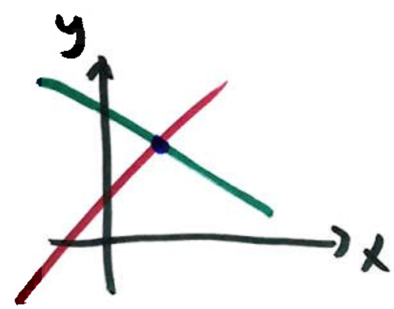
from ①: $3y = 9 - x$

$$y = 3 - \frac{1}{3}x$$

$$= 3 - \frac{1}{3}(3)$$

$$\boxed{y = 2}$$

these two lines intersect once at (3, 2)



<u>example</u>

$$x + 2y = 4 \quad - \text{①}$$
$$2x + 4y = 9 \quad - \text{②}$$

to eliminate x, $-2\text{①} + \text{②}$

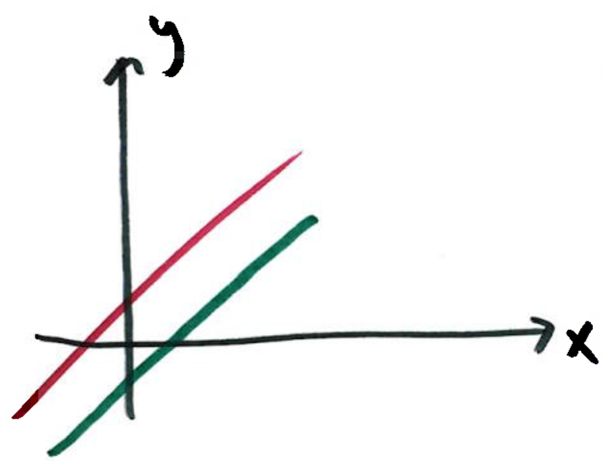
$$\left.\begin{array}{ll} -2x - 4y = -8 & (-2\text{①}) \\ 2x + 4y = 9 & (\text{②}) \end{array}\right\} \text{add}$$

$0 = 1$ clearly false

no (x, y) works in BOTH eqs

→ lines are parallel

(no intersection)



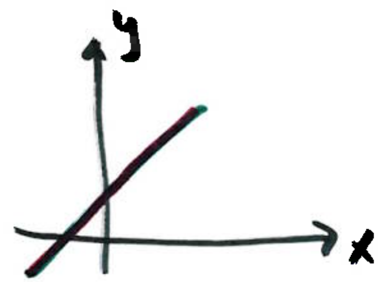
<u>Example</u>

$$x + 2y = 4 \quad - ①$$

$$2x + 4y = 8 \quad - ②$$

$$-2① + ② \to \quad 0 = 0$$

clearly true regardless of (x, y)



ANY (x, y) on one line also works on the other

two identical lines

infinitely-many solutions

to express the solutions, pick one variable at will, use either equation to solve the other

let $y = t$ (whatever we want)

then ① : $x = 4 - 2y = 4 - 2t$

solutions: $(4-2t, t)$ or $x = t$, ① : $y = 2 - \frac{1}{2}t$

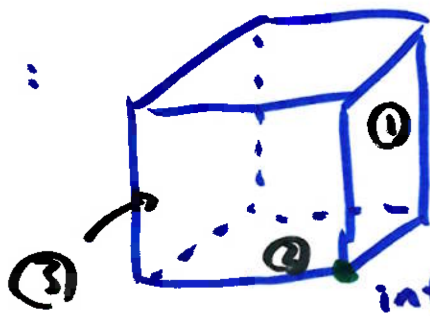
$(t, 2 - \frac{1}{2}t)$

3D: 3 equations w/ 3 variables

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

} 3 planes

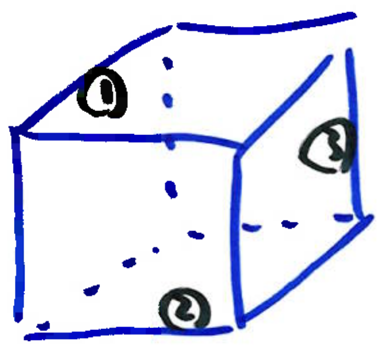
possibilities :



one solution

intersection of planes 1, 2, 3

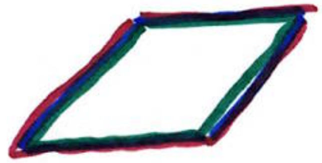
(intersection of ceiling and two walls)
or floor



ceiling, floor, one wall

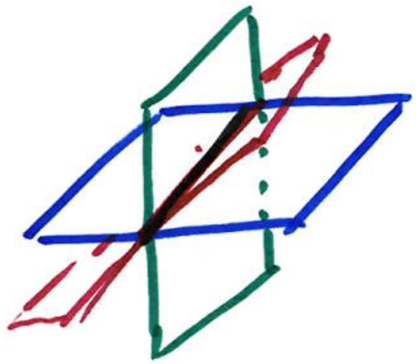
two planes intersect
but the 3rd one
only intersects one

no solution



all 3 are the same

infinitely-many solutions



3 planes intersect at one ~~p~~ line

infinitely-many solutions

all parallel

no solution

Solution method : elimination (same idea)

example

$$x + 5y + z = 2 \quad - (1)$$
$$2x + y - 2z = 1 \quad - (2)$$
$$x + 7y + 2z = 3 \quad - (3)$$

work w/ two at a time to eliminate a variable

for example, let's eliminate x from (1), (2)

$-2①+② \longrightarrow -9y-4z=-3$ — ④

$-1①+③ \longrightarrow 2y+z=1$ — ⑤

} two eqs. two variables (2D)

$+4⑤+④ \qquad -y=1 \rightarrow \boxed{y=-1}$

⑤: $z=1-2y=1-2(-1) \rightarrow \boxed{z=3}$

①: $x=2-5y-z$

$=2-5(-1)-(3)$

$\boxed{x=4}$

one solution

three planes meet at one point