3.4 Matrix Operations

notation: usually capital letters A, B, etc

in print (books) often bold faced

each number in a matrix is called an element identified by the row and column it occupies

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

elements are usually the lowercase equivalent

$a_{11} = 1 \qquad a_{12} = 2 \qquad a_{13} = 3$

row column

$a_{23} = 6$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = [a_{ij}]$$

compact way to express matrix

Addition : we can add two matrices of the same dimension
(# rows and # columns)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & -2 \\ -5 & -10 \end{bmatrix}$$

2×2 $\qquad$ 2×3

$$A + B = \begin{bmatrix} 1 + (-1) & 2 + (-2) \\ 3 + (-5) & 4 + (-10) \end{bmatrix}$$ element-by-element

$$= \begin{bmatrix} 0 & 0 \\ -2 & -6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

2×3 $\qquad$ 3×2

$A + C$ is not possible (different sizes)

neither is $A + C$ or $C + D$

Scalar multiplication: scalar c (normally lowercase)

matrix A

then $cA = [c \cdot a_{ij}]$ multiply each element by scalar c

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \end{bmatrix}$$

$$5A = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} \qquad 5B = \begin{bmatrix} 5 & 10 & 15 \\ 30 & 25 & 20 \end{bmatrix}$$

$$-3A = \begin{bmatrix} -3 & -6 \\ -9 & -12 \end{bmatrix} \qquad -3B = \begin{bmatrix} -3 & -6 & -9 \\ -18 & -15 & -12 \end{bmatrix}$$

Subtraction: ~~A+~~ $A - B = A + (-1)B$

same size

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -1 & -5 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + (-1)\begin{bmatrix} 0 & -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 8 \end{bmatrix}$$

matrices w/ one row or one column only are called <u>vectors</u>

$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ is a <u>row vector</u>

$C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a <u>column vector</u>

<u>Matrix multiplication</u> : very different from numbers

AB is possible only if # columns of A is equal to # rows of B

first matrix has the same # of columns as the # of rows of the second matrix

result is a matrix w/ # of rows of 1st and # of columns of 2nd

$$\underset{\boxed{3}\times\boxed{3\quad 3}\times\boxed{1}}{A\,B} = \underset{3\times 1}{C}$$

must match

rows of result

columns of result

<u>example</u>

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & 0 & 3 \\ 1 & -5 & 2 \end{bmatrix}$$

$$2\times 2 \qquad\qquad 2\times 3$$

$$\underset{2\times\boxed{2\quad 2}\times 3}{A\,B} = \begin{bmatrix} \boxed{1 \quad 1} \\ 2 \quad 1 \end{bmatrix} \begin{bmatrix} \boxed{\begin{matrix}-4\\1\end{matrix}} & \boxed{\begin{matrix}0\\-5\end{matrix}} & \boxed{\begin{matrix}3\\2\end{matrix}} \end{bmatrix}$$

match!

→ mult. then add

$$= \begin{bmatrix} 1\cdot -4 + 1\cdot 1 & 1\cdot 0 + 1\cdot -5 & 1\cdot 3 + 1\cdot 2 \\ 2\cdot -4 + 1\cdot 1 & 2\cdot 0 + 1\cdot -5 & 2\cdot 3 + 1\cdot 2 \end{bmatrix}.$$

$$= \begin{bmatrix} -3 & -5 & 5 \\ -7 & -5 & 8 \end{bmatrix}$$

BA is NOT possible

$2\times\boxed{3 \quad 2}\times 2$

do NOT match

$$\boxed{\text{for matrices, } AB \neq BA \text{ in general}}$$

example

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 9 \\ -5 & -6 \end{bmatrix}$$

2×2 $\qquad$ 2×2

AB and BA are both possible

$$AB = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 9 \\ -5 & -6 \end{bmatrix} = \begin{bmatrix} -7 & 12 \\ -6 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 9 \\ -5 & -6 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ -16 & -11 \end{bmatrix}$$

$AB \neq BA$ in general

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$2 \times 2 \qquad 2 \times 2$

$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

this is one exception to $AB \neq BA$

because the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a 2×2 <u>identity matrix</u>

it is the matrix equivalent of the number 1 : $1 \cdot 5 = 5 \cdot 1$

$1 \cdot 10 = 10 \cdot 1$

there is NO matrix division but there is matrix inverse (later part of chapter)

→ equivalent to scalar inverse: $1 \cdot a^{-1} = \frac{1}{a}$

ways to interpret linear systems

$$x_1 + 3x_2 = 9$$
$$2x_1 + x_2 = 8$$

express as a matrix equation:

$$\underbrace{\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 9 \\ 8 \end{bmatrix}}_{\vec{b}}$$

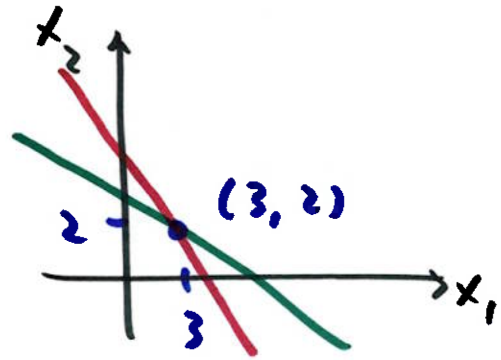
express as a vector equation:

$$x_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

1st interpretation of

$$\begin{aligned} x_1 + 3x_2 &= 9 \\ 2x_1 + x_2 &= 8 \end{aligned}$$

is intersection of the two lines



2nd interpretation:

$$x_1\begin{bmatrix}1\\2\end{bmatrix} + x_2\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}9\\8\end{bmatrix}$$