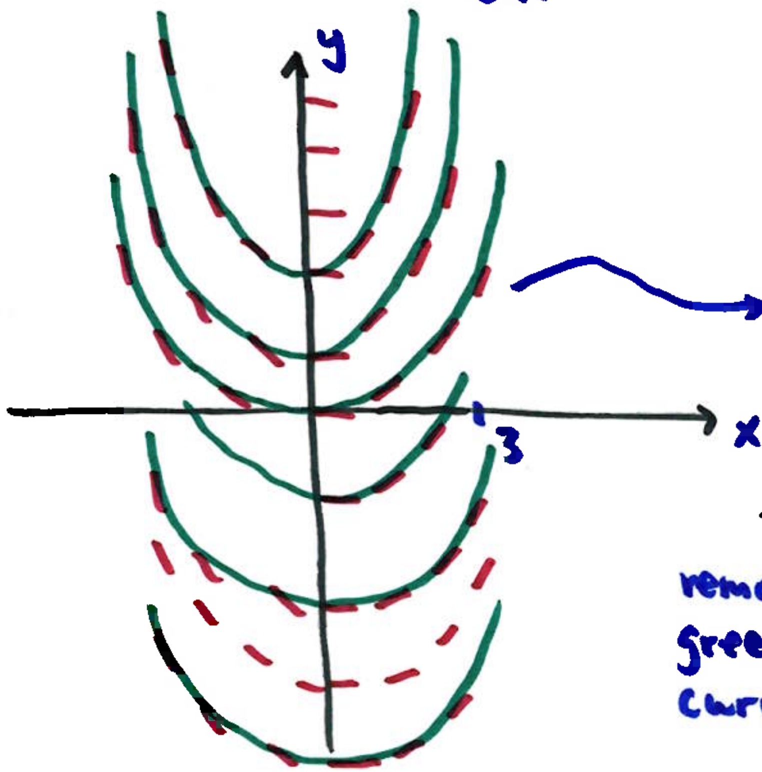
# 1.3 Slope Field and Solution Curves

a qualitative way to analyze solutions

from last time: $\frac{dy}{dx} = 2x$ solution $y = x^2 + C$




tangent lines at different places

each has slope given by $\frac{dy}{dx} = 2x$

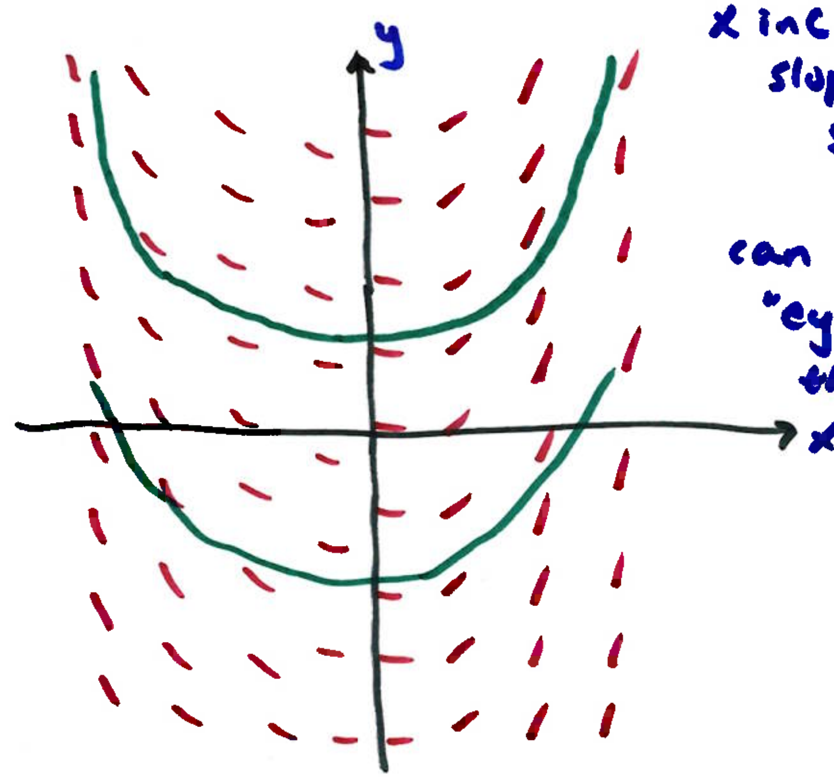
slope = 2(3) = 6

x inc

slope

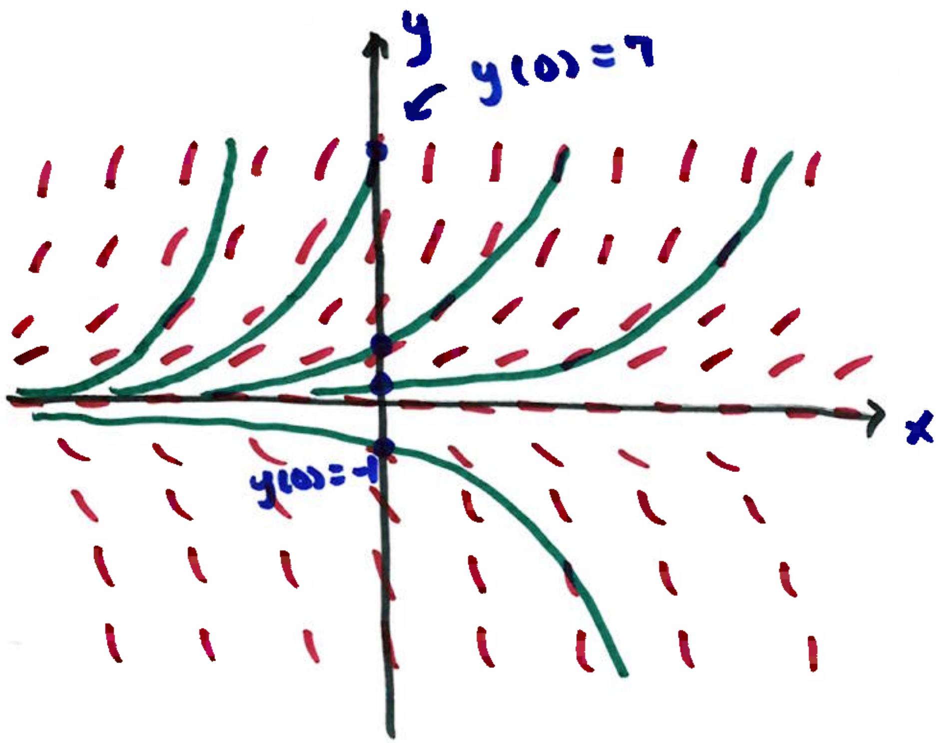
steeper

remove green curves




can "eyeball" the solution curves

<u>example</u> $\frac{dy}{dx} = y$




each curve corresponds to a different initial condition

$\underbrace{\frac{dy}{dx}}_{\text{slope}} = y$

↳ does not depend on $x$

for given $y$ entire row of equal slopes

$y=0 \to$ zero slopes

as $y \to \infty$ slope increases

$y \to -\infty$ increasing negative slopes

the solution curves look exponential

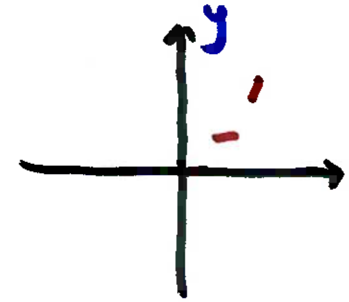
(and they are indeed exponential)

**example** $\frac{dy}{dx} = \underbrace{y - x}$

at point $(x, y)$ slope is $y - x$

each point has potentially different slope

picking points is not efficient



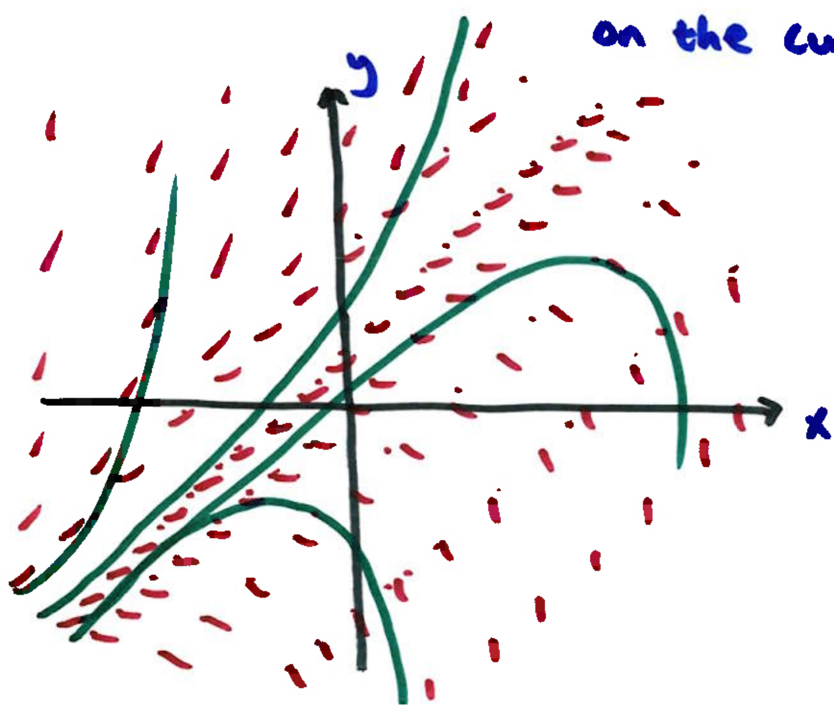

$(0,0) \to 0$

$(1,1) \to 0$

$(1,2) \to 1$

keep doing this?

better way: $\frac{dy}{dx} = y - x$



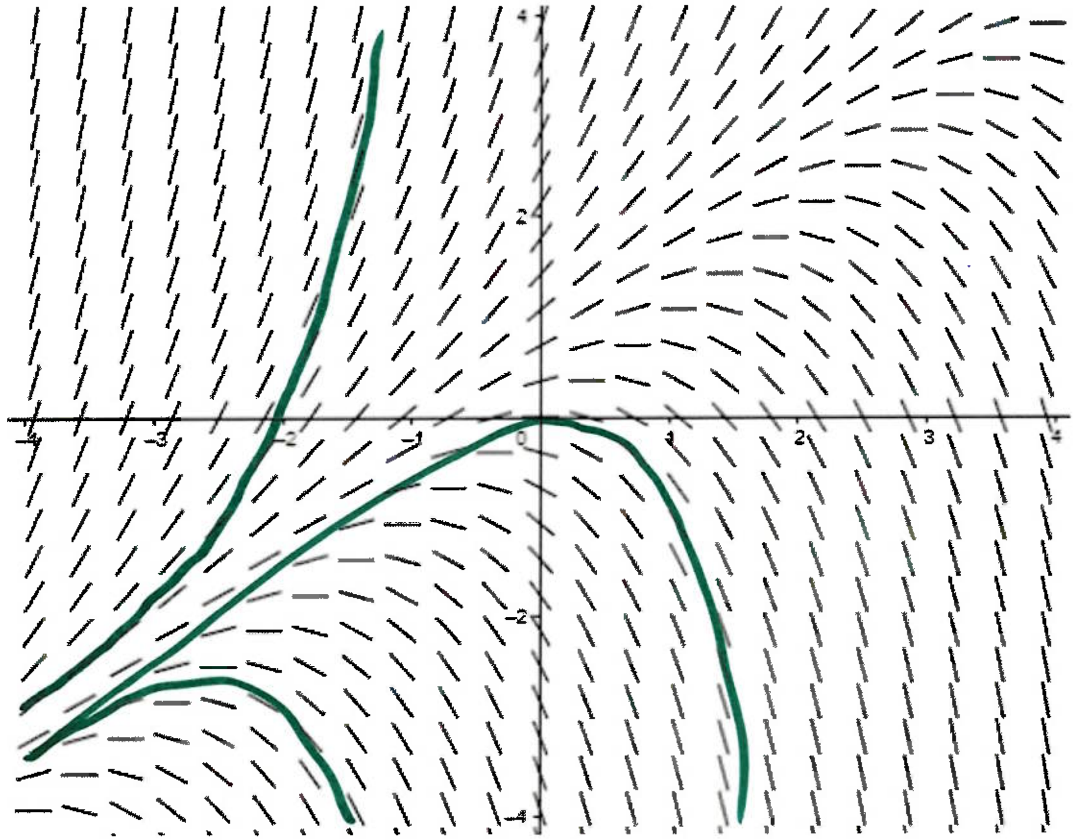

on the curve $y = x$ slopes are zero

above $y = x \to y - x > 0 \to$ pos. slopes

higher above $\to$ greater slope

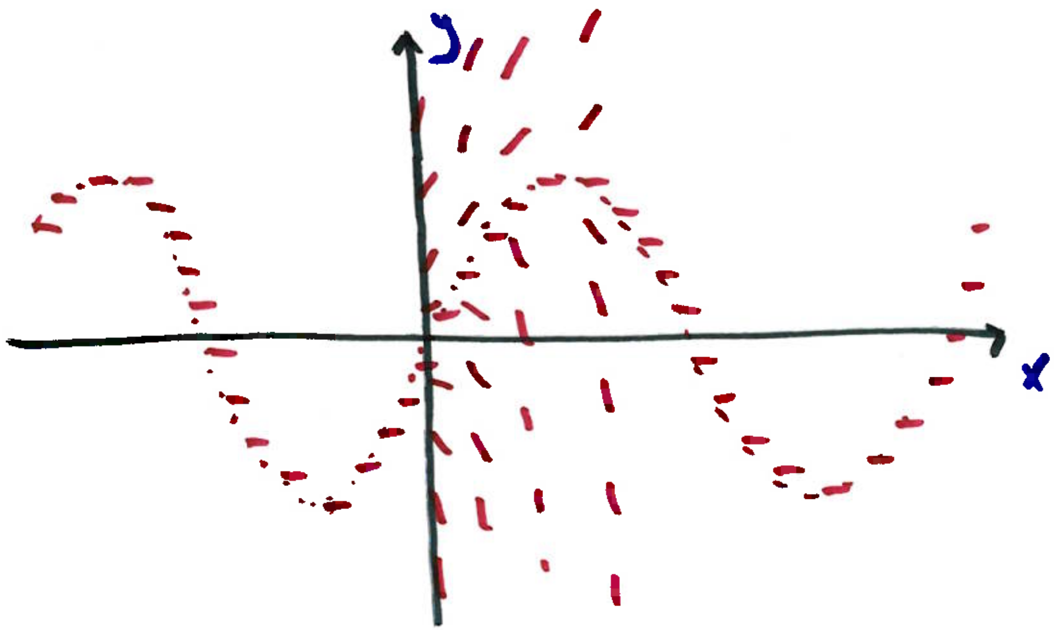
below $\to$ negative slopes

$$\frac{dy}{dx} = y - x$$



<u>example</u> $y' = y - \sin x$

$y - \sin x = 0 \to$ zero slopes so on $y = \sin x$ slopes are zero




fix $x$ and inc $y$

slopes increase

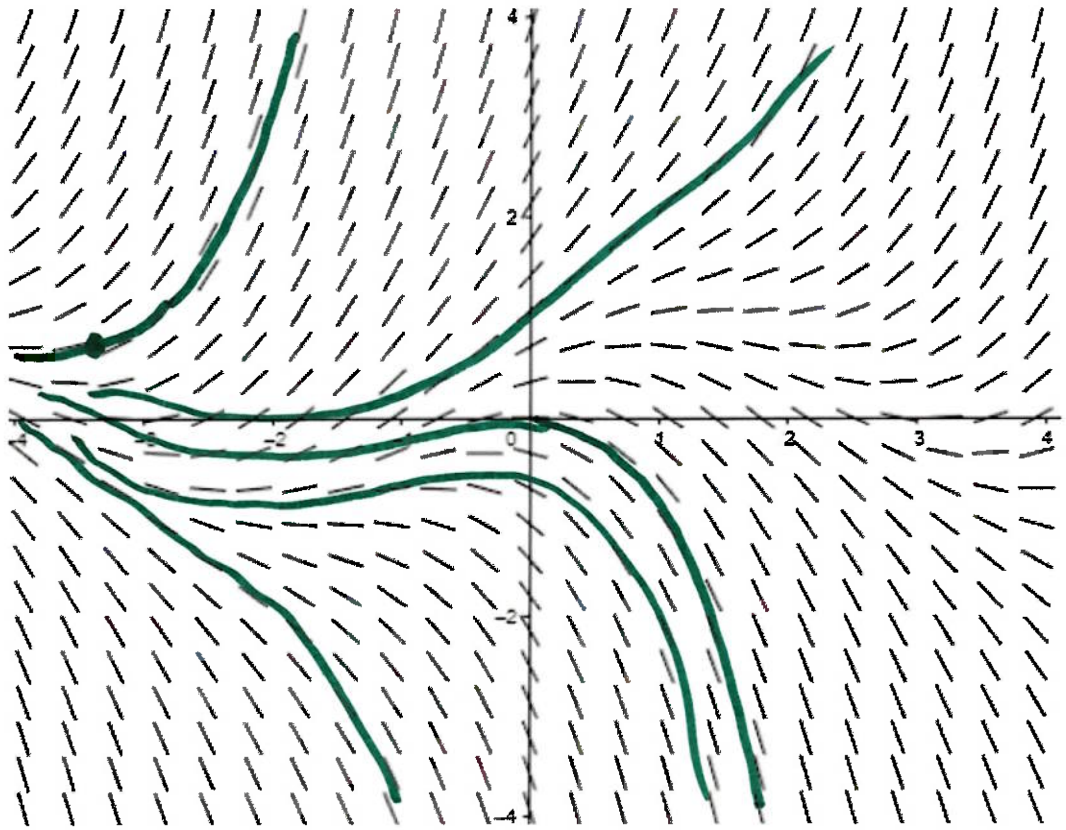
fix $x$ and dec $y$

slopes more negative

fix $y$ and inc $x$

or fix $y$ and dec $x$

same idea



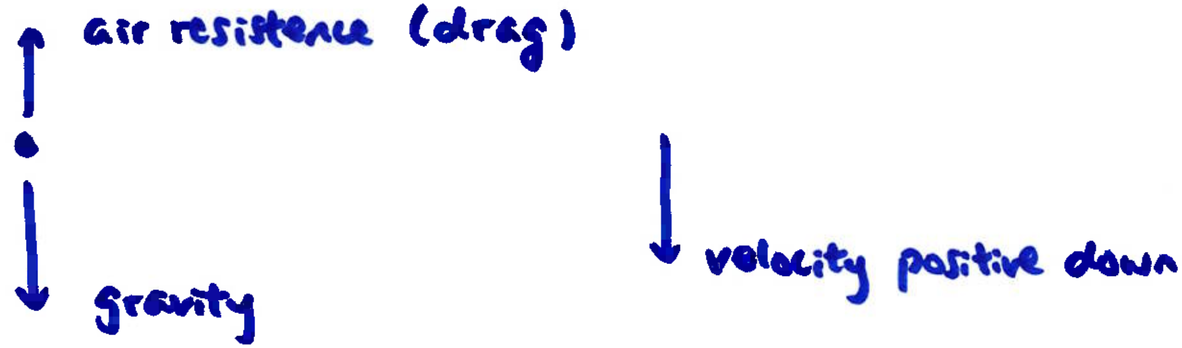

Initial y at $x=0$

determines long term

behavior if $y(0) > 0.52$ $y \to \infty$

$y(0) < 0.52$ $y \to -\infty$

application: terminal velocity




Newton's 2nd Law: $F = ma = m\frac{dv}{dt}$ v: velocity

$$m\frac{dv}{dt} = \underbrace{mg}_{\text{weight}} - \underbrace{d}_{\text{drag}}$$

$$\boxed{\frac{dv}{dt} = g - \frac{d}{m}}$$

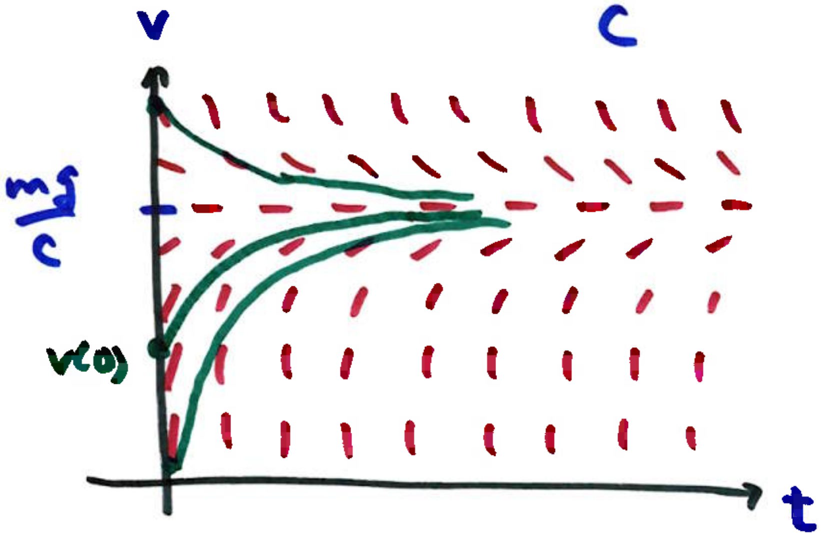
differential equation

drag can be modeled as $d = \overset{\text{drag coefficient}}{c}\,v$

$$\frac{dv}{dt} = g - \frac{cv}{m} = g - \frac{c}{m}v$$

Slope field: when $g - \frac{c}{m}v = 0 \to$ zero slope

$$v = \frac{mg}{c}$$




above: slope is negative same for all t

below: opposite

if $v(0) < \frac{mg}{c}$, pick up speed then approach $\frac{mg}{c}$

if $v(0) > \frac{mg}{c}$, slow down then approach $\frac{mg}{c}$

large c → parachute → slow terminal velocity (survival!)

Small c → opposite