7.4 Solution Curves of Linear Systems (pa

$\vec{x}' = \begin{bmatrix} 4 & 7 \\ 7 & 4 \end{bmatrix} \vec{x}$ $\lambda = -3, 11$

$\vec{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

solution: $\vec{x}(t) = c_1 e^{-3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^{11t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$

$x_1(t) = -c_1 e^{-3t} + c_2 e^{11t}$

$x_2(t) = c_1 e^{-3t} + c_2 e^{11t}$

graph of $x_1(t)$ vs. $x_2(t)$ is called a phase

gives relationship (qualitatively) between

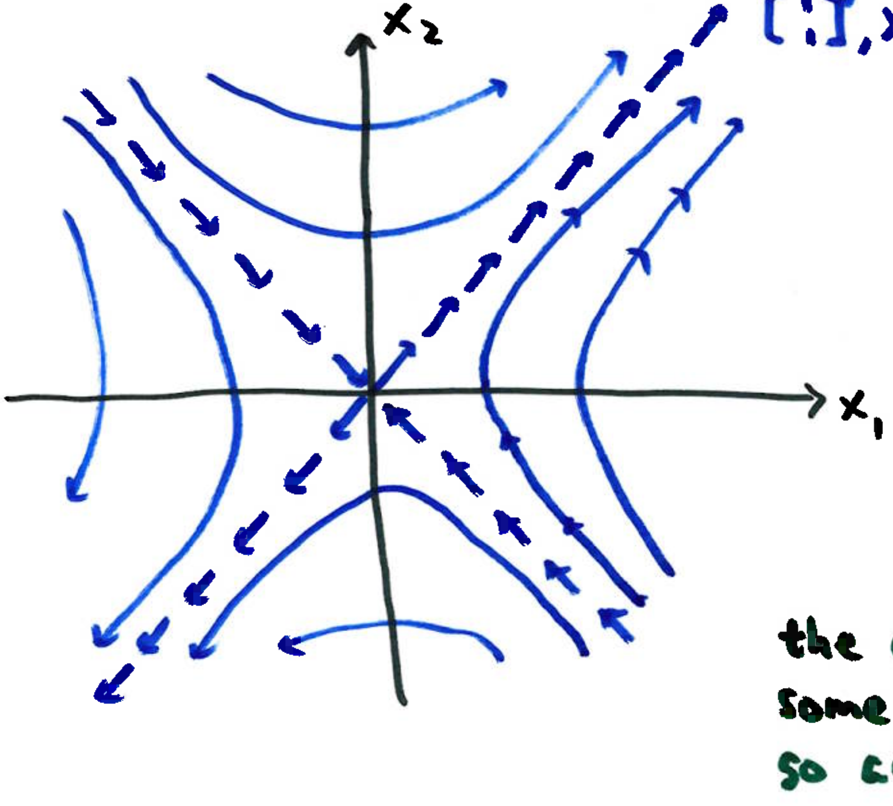
x_1, x_2 as t changes

$$\vec{x} = \underbrace{c_1 e^{-3t}}_{\substack{\text{dominates} \\ \text{as } t \to -\infty}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \underbrace{c_2 e^{11t}}_{\text{dominate as } t \to \infty} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

as $t \to \infty$, all solutions approach the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$t \to -\infty$ " " " " $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

these vectors behave like asymptotes



$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \lambda > 0$ if initial condition

$c_1 = 0$, then $\vec{x} = c$

then $x_1 \to \infty$, x_2

(because $\lambda > 0$)

if $c_2 < 0$, then t

for the other, foll

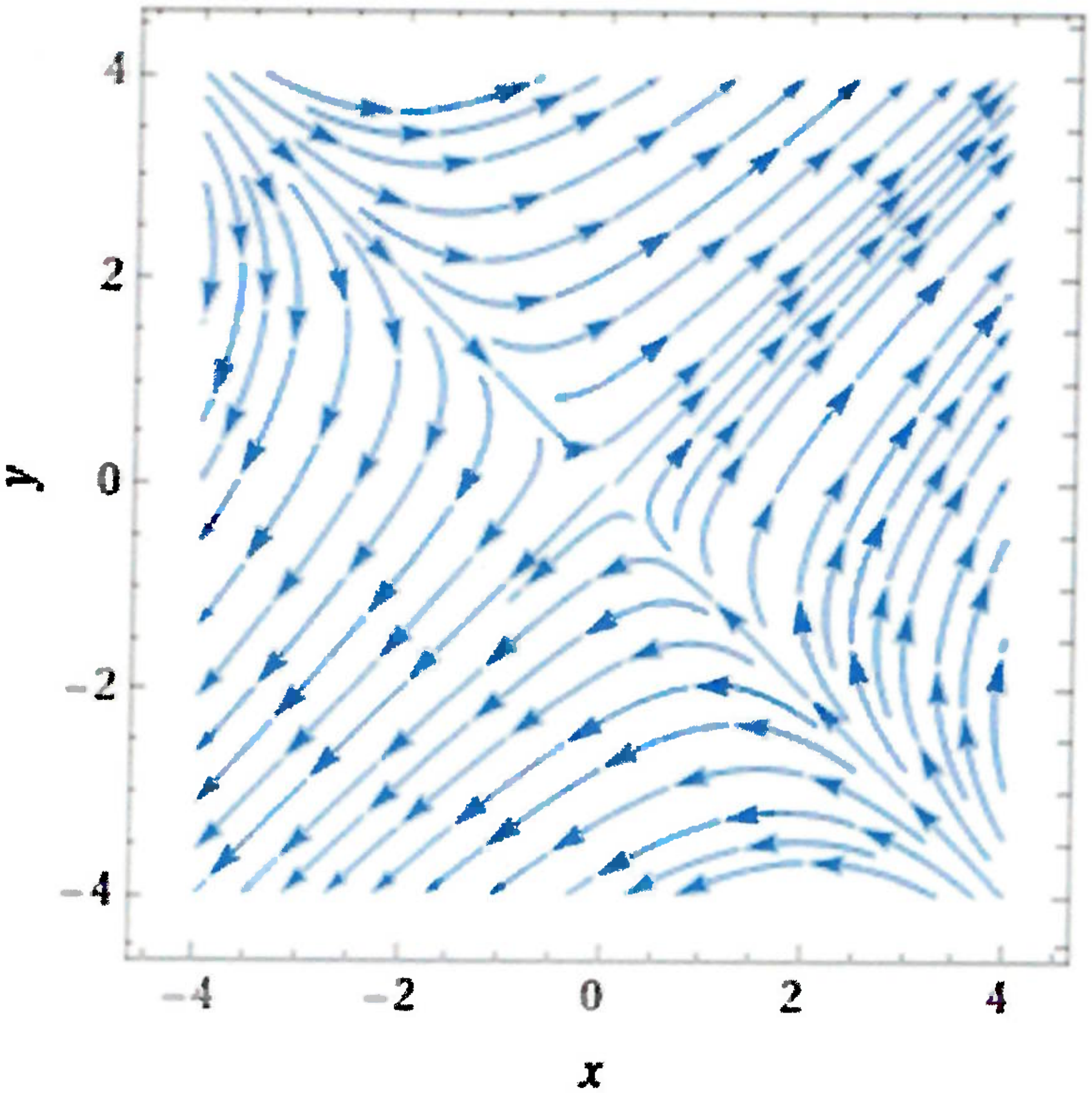
reasoning, soluti

origin because λ

the origin here is called a

some solutions go toward orig

go away → λ's are <u>opposi</u>


4
2
0
−2
−4
y
−4
−2
0
2
4
x

example $\vec{x}' = \begin{bmatrix} 6 & 5 \\ -3 & -2 \end{bmatrix} \vec{x}$

$\lambda = 3, \ 1$

$\vec{v} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$\vec{x} = \underbrace{c_1 e^{3t} \begin{bmatrix} -5 \\ 3 \end{bmatrix}}_{\text{dominates as } t \to \infty} + c_2 e^{t} [$

do

fo

so, solutions will

first following [

as t → ∞, follo

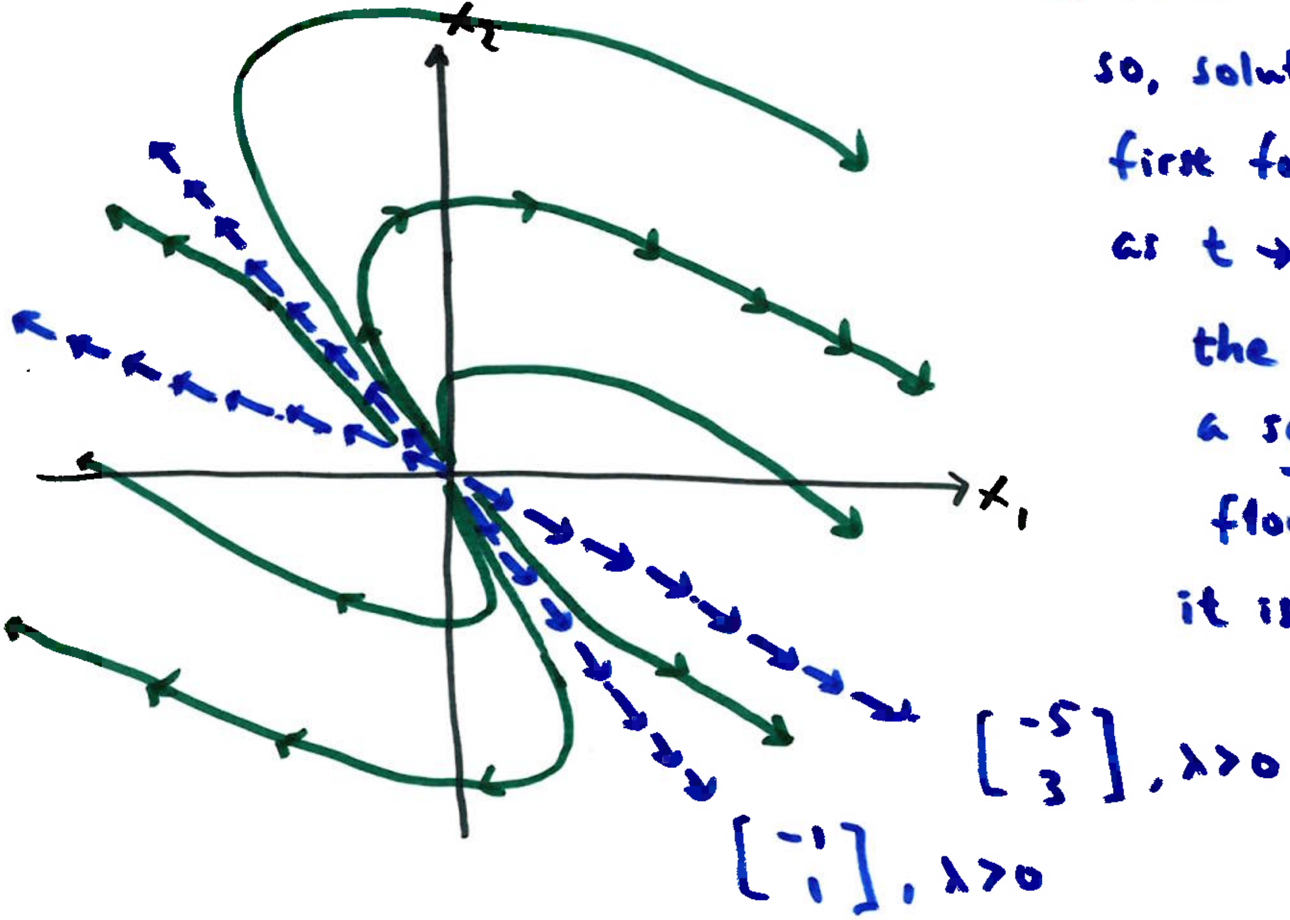
the origin is

a source be

flow out of

it is an imp

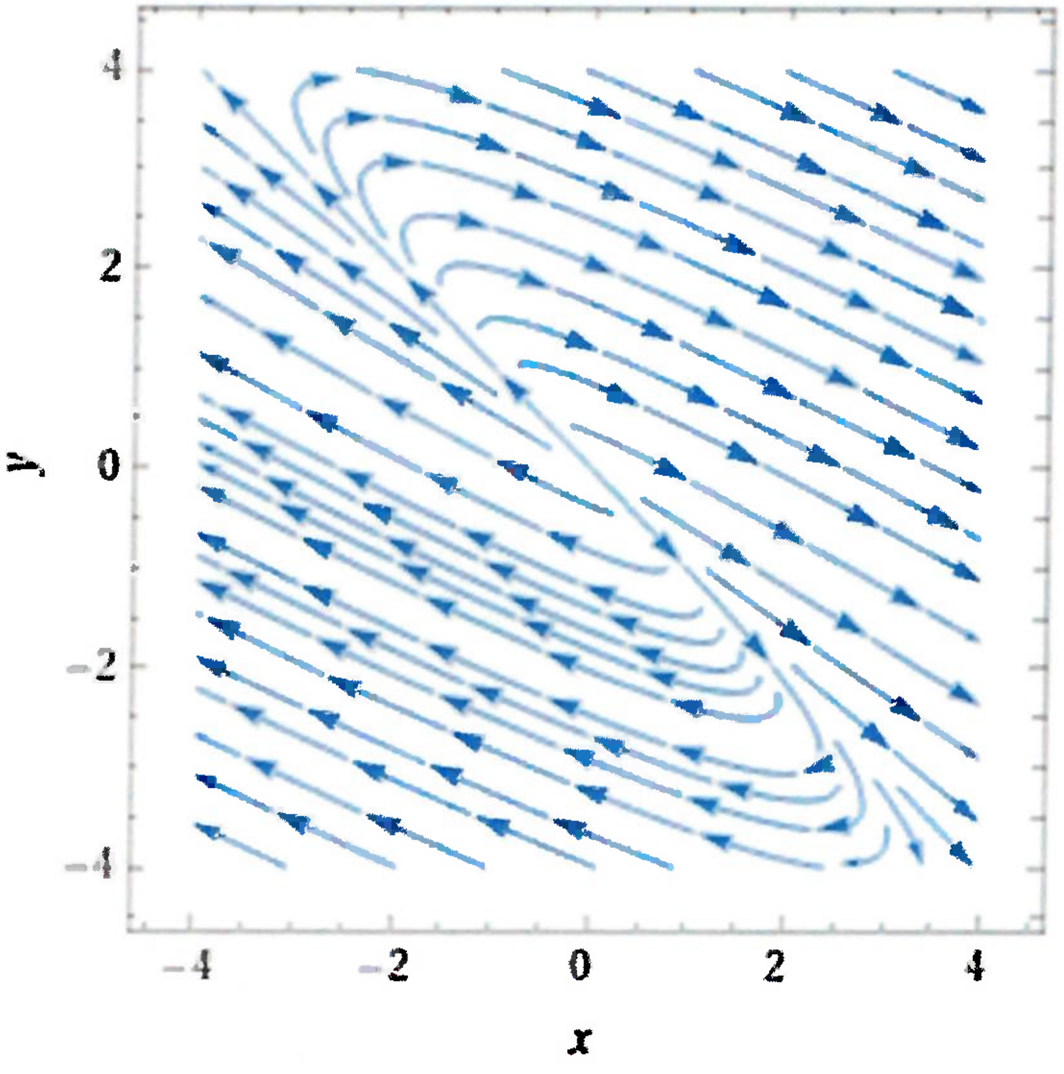
source



s

f

as



4
2
0
−2
−4
y
−4
−2
0
2
4
x

if both λ's are negative, the same (or similar) pictu

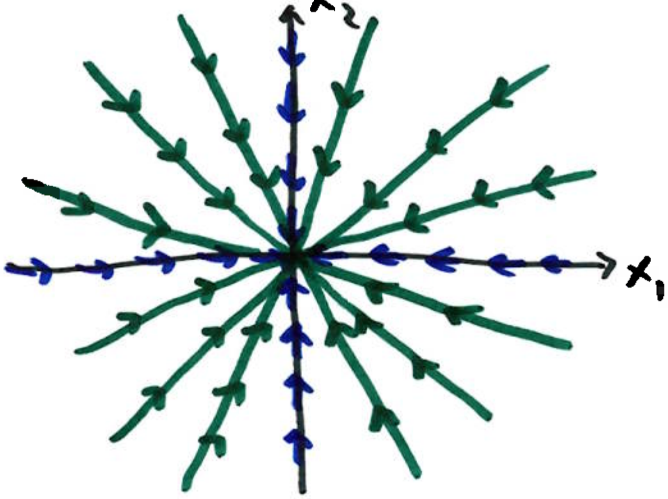
but all arrows go toward origin → improper nodal s

example $\vec{x}' = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \vec{x}$ $\qquad \lambda = -1, -1$

$\vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\vec{x} = \underbrace{c_1 e^{-t}}_{\cancel{x_1}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$x_1 = c_1 e^{-t}$ $\qquad x_2 = c_2 e^{-t}$ $\qquad \frac{x_2}{x_1} = \frac{c_2}{c_1} = m$ →



origin is a pro

↓

no asymptotes

repeated λ's

w/ enough

eigenvectors

(or m

example

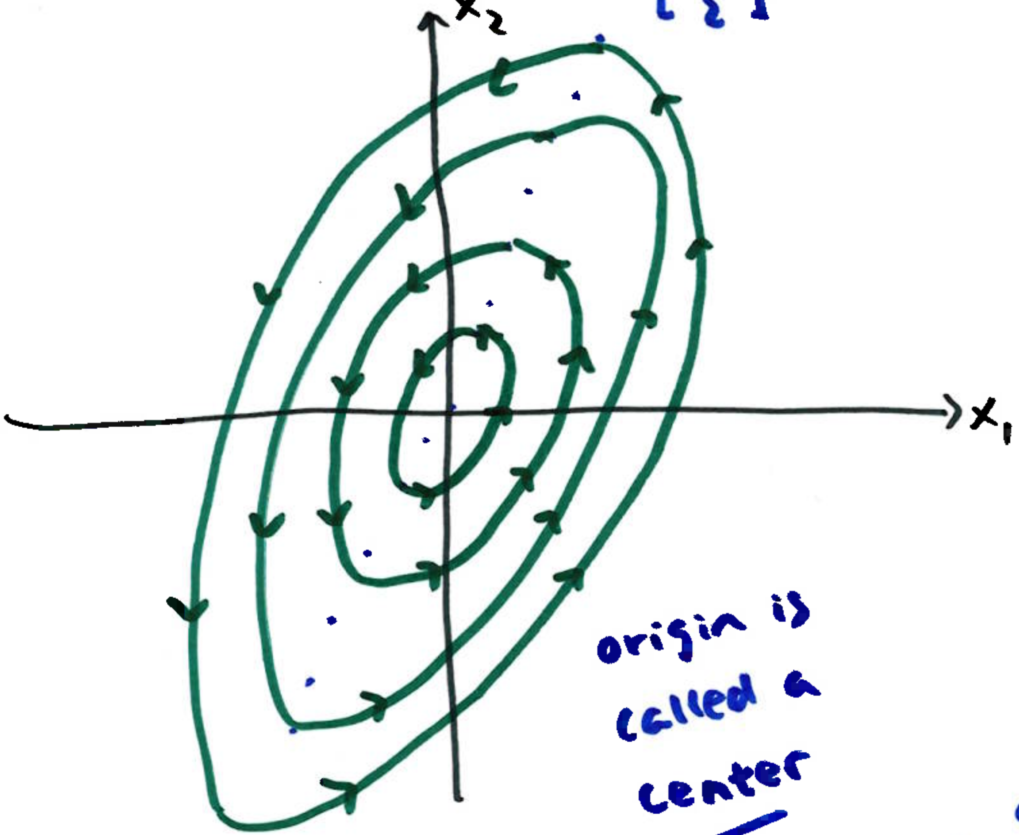
$$\vec{x}' = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \vec{x}$$

$\lambda = i, -i$

$\vec{v} = \begin{bmatrix} 1+i \\ 2 \end{bmatrix}, \begin{bmatrix} 1-i \\ 2 \end{bmatrix}$

$$\vec{x} = c_1 \begin{bmatrix} \sin t + \cos t \\ 2 \sin t \end{bmatrix} + c_2 \begin{bmatrix} -\sin t \\ \cos t - \sin t \end{bmatrix}$$

sine, cosine → periodic → rotation (solutions a
$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ that go



origin is called a **center**

real part of

the major

which directi

CCW or C

pick a loca

example, $\vec{x}$=

then $\vec{x}' = A\vec{x}$

$\vec{x}' = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

tangent vector

example

$$\vec{x}' = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \vec{x}$$

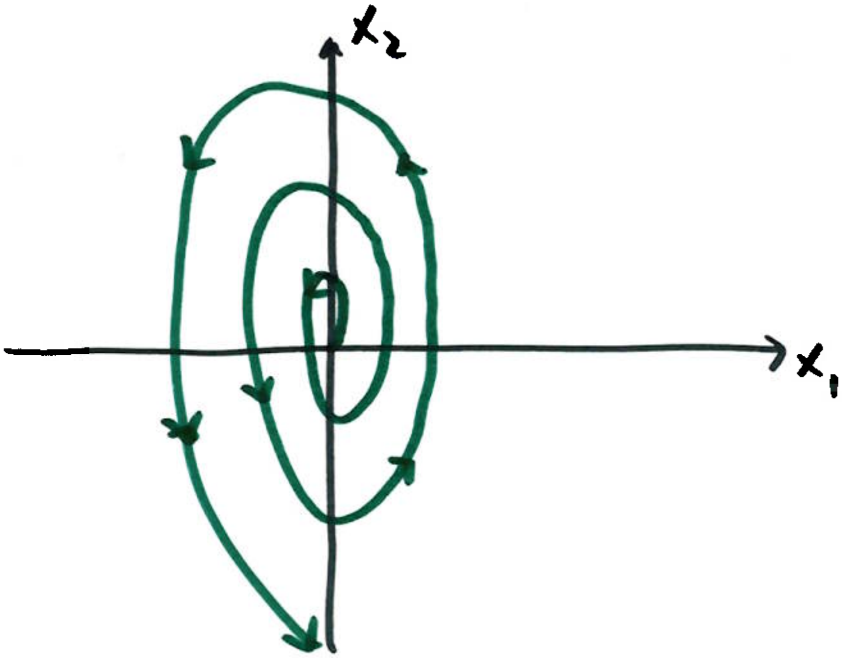
$\lambda = 1-i,\ 1+i$

$\vec{v} = \begin{bmatrix} i \\ 1 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \end{bmatrix}$

$\vec{x} = c_1 e^t \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix} + c$

direction determined as in last example

due to positi
of λ, incre
so the curves
from origin
spiral



origin here
a spiral so

(if real part
is <0, spi