

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

- (1) (10 Points) Use Cramer's rule to compute the solutions of the system

$$\begin{cases} -5x_1 + 2x_2 = 9 \\ 3x_1 - x_2 = -4 \end{cases}$$

Answer: We shall have

$$A_1 = \begin{bmatrix} 9 & 2 \\ -4 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} -5 & 9 \\ 3 & -4 \end{bmatrix}, A = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$

Thus, we have

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{-1}{-1} = 1,$$
$$x_2 = \frac{\det(A_2)}{\det(A)} = \frac{-7}{-1} = 7.$$

- (2) (10 Points) Use determinants to find out if the set of vectors is linearly independent.

$$\begin{bmatrix} 7 \\ -4 \\ -6 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 5 \end{bmatrix}.$$

Answer: Consider the matrix

$$A = \begin{bmatrix} 7 & -8 & 7 \\ -4 & 5 & 0 \\ -6 & 7 & 5 \end{bmatrix}$$

The determinant of A is $\det A = 29$, that means matrix A is invertible. Therefore the three given vectors are linearly independent.