

Name: _____

ID number: _____

Instructions:

1. This is a one-hour exam.
2. There are 8 multiple-choice problems, 4 free-response problems, and 10 true-or-false problems.
3. No books, notes, or calculators are allowed.
4. Please turn off your cell phone.
5. Circle one and only one choice for each multiple-choice problem. No partial credit will be given for multiple-choice problems.
6. Show all relevant work on free-response problems. Partial credit will be given for steps leading to the correct solutions.
7. For each correct answer in the true-or-false section you will earn 2 points. You will lose 1 point for each incorrect answer. No points will be awarded or deducted for not answering a question. You are not required to show work for true-or-false problems.

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

I agree to abide by the instructions above:

Signature: _____

1. (6 points) Below is an echelon form of an unknown matrix A . What are the dimensions of the column space and null space of A ?

$$\begin{bmatrix} 1 & 4 & 4 & 6 \\ 0 & 0 & 7 & -6 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- A. $\dim \text{Col } A=2$, $\dim \text{Nul } A=2$
- B. $\dim \text{Col } A=4$, $\dim \text{Nul } A=0$
- C. $\dim \text{Col } A=0$, $\dim \text{Nul } A=4$
- D. $\dim \text{Col } A=3$, $\dim \text{Nul } A=1$
- E. $\dim \text{Col } A=1$, $\dim \text{Nul } A=3$

2. (6 points) If $\begin{vmatrix} a & b & c \\ 9 & 4 & 6 \\ 5 & 2 & 8 \end{vmatrix} = 6$, what is the value of $\begin{vmatrix} 9 & 4 & 6 \\ a & b & c \\ 10 & 4 & 16 \end{vmatrix}$?

- A. -12
- B. -6
- C. -3
- D. 3
- E. 12

3. (6 points) Find the volume of the parallelopiped with edges defined by

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} -1 \\ 5 \\ 3 \end{bmatrix}, \text{ and } \vec{w} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$$

- A. 0
- B. 1
- C. $\sqrt{2}$
- D. $\sqrt{10}$
- E. 10

4. (6 points) If A is a 3×3 matrix and

$$A \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} = \vec{0}, \quad A \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -6 \end{bmatrix}, \text{ and } \quad A \begin{bmatrix} -1 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 5 \\ -25 \\ 35 \end{bmatrix}$$

then an eigenvalue of A is

- A. 0
- B. -2
- C. 2
- D. -5
- E. 5

5. (6 points) Suppose A is an 8×7 matrix with a null space dimension of 6, then the rank of A is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

6. (6 points) Find the dimension of the subspace spanned by

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix} \right\}$$

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4

7. (6 points) If A is a 5×3 matrix, what is the largest possible dimension of $\text{Row } A$?

- A. 2
- B. 3
- C. 4
- D. 5
- E. Depends on what A looks like

8. (6 points) Let $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Find the dimension of the eigenspace of A corresponding to the eigenvalue $\lambda = 1$

- A. 1
- B. 2
- C. 3
- D. 4
- E. 0

9. (8 points) The vector \vec{x} is in a subspace H with basis $B = \{\vec{b}_1, \vec{b}_2\}$. Find the B -coordinate vector of \vec{x}

$$\vec{b}_1 = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} -2 \\ -7 \\ 5 \end{bmatrix}, \vec{x} = \begin{bmatrix} -5 \\ -17 \\ 12 \end{bmatrix}$$

10. (8 points) Find all eigenvalues of

$$A = \begin{bmatrix} 3 & 0 & 5 \\ 0 & 3 & 0 \\ 1 & 0 & 7 \end{bmatrix}$$

11. (8 points) Diagonalize, if possible.

$$\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$$

12. (8 points) Let $M_{2 \times 2}$ be the vector space of all 2×2 matrices, and define $T : M_{2 \times 2} \rightarrow M_{2 \times 2}$ by $T(A) = A + A^T$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Is T a linear transformation? Fully justify your answer.

13. Circle at most one response for each question. (2 pts each if correct, -1 if incorrect, 0 if no response)

(a). If H is a p -dimensional subspace of \mathbb{R}^n , then a linearly independent set of p vectors in H is a basis for H .

TRUE FALSE

(b). $\det(A + B) = \det(A) + \det(B)$.

TRUE FALSE

(c). A subspace is also a vector space.

TRUE FALSE

(d). The kernel of a linear transformation is a vector space.

TRUE FALSE

(e). The columns of an invertible $n \times n$ matrix form a basis for \mathbb{R}^n .

TRUE FALSE

(f). The number of variables in the equation $A\vec{x} = \vec{0}$ equals the dimension of $\text{Nul } A^T$.

TRUE FALSE

(g). Row operations preserve the linear dependence relations among the rows of a matrix.

TRUE FALSE

(h). $\vec{0}$ can be an eigenvector.

TRUE FALSE

(i). If a 2×2 matrix has eigenvalues 1 and 3, then the matrix is invertible.

TRUE FALSE

(j). A square matrix is diagonalizable if it can be row reduced to a diagonal matrix.

TRUE FALSE