

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

- (1) (10 Points) For the following matrix

$$A = \begin{bmatrix} 0 & 1 \\ -8 & 4 \end{bmatrix}$$

- 1) find an eigenvalue  $\lambda$  and a corresponding eigenvector  $\mathbf{v}$ .

- 2) find an invertible matrix  $P$  and a matrix  $C$  of the form  $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$  such that given matrix has the form  $A = PCP^{-1}$ .

- (2) (10 Points) Let  $A$  be a  $2 \times 2$  matrix with eigenvalues  $-3$  and  $1$  and corresponding eigenvectors  $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$  and  $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ . Let  $\mathbf{x}(t)$  be the position of a particle at time  $t$ , solve the initial value problem  $\mathbf{x}' = A\mathbf{x}$ ,  $\mathbf{x}(0) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ .