

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

- (1) (10 Points) Write down the augmented matrix of the following system, and is $(1, 0, 1)$ is a solution of it?

$$\begin{cases} x_1 + 2x_2 + 3x_3 &= 4 \\ 2x_1 - x_2 + x_3 &= 3 \\ -x_1 + x_2 + x_3 &= 0 \end{cases}$$

Answer:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & -1 & 1 & 3 \\ -1 & 1 & 1 & 0 \end{pmatrix}$$

Yes. To verify if $(1, 0, 1)$ is a solution of this system, we just substitute it back to these equations:

$$1 + 2 * 0 + 3 * 1 = 4,$$

$$2 * 1 - 0 + 1 = 3,$$

$$-1 + 0 + 1 = 0.$$

Therefore, it's a solution of this system.

- (2) (10 Points) Find an equation involving g , h , and k that makes this augmented matrix correspond to a consistent system:

$$\begin{pmatrix} 1 & 0 & 1 & g \\ 2 & 6 & 0 & h \\ -1 & 3 & -2 & k \end{pmatrix}$$

Answer: $4g - h + 2k = 0$.

By $R_2 - 2R_1$, we get

$$6x_2 - 2x_3 = h - 2g.$$

Similarly, by $R_1 + R_3$, we will have

$$3x_2 - x_3 = g + k.$$

Since we require it to be a consistent system, we need

$$h - 2g = 6x_2 - 2x_3 = 2 * (3x_2 - x_3) = 2 * (g + k)$$

which is $4g - h + 2k = 0$.