

Name: _____

ID number: _____

Instructions:

1. This is a one-hour exam.
2. There are 7 multiple-choice problems, 4 free-response problems, and 9 true-or-false problems.
3. No books, notes, or calculators are allowed.
4. Please turn off your cell phone.
5. Circle one and only one choice for each multiple-choice problem. No partial credit will be given for multiple-choice problems.
6. Show all relevant work on free-response problems. Partial credit will be given for steps leading to the correct solutions.
7. For each correct answer in the true-or-false section you will earn 2 points. You will lose 1 point for each incorrect answer. No points will be awarded or deducted for not answering a question. You are not required to show work for true-or-false problems.

Purdue University faculty and students commit themselves towards maintaining a culture of academic integrity and honesty. The students taking this exam are not allowed to seek or obtain any kind of help from anyone to answer questions on this test. If you have questions, consult only an instructor or a proctor. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you finish your exam and hand it in to a proctor or to an instructor. You may not consult notes, books, calculators, cameras, or any kind of communications devices until after you finish your exam and hand it in to a proctor or to an instructor. If you violate these instructions you will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students. Your instructor and proctors will do everything they can to stop and prevent academic dishonesty during this exam. If you see someone breaking these rules during the exam, please report it to the proctor or to your instructor immediately. Reports after the fact are not very helpful.

I agree to abide by the instructions above:

Signature: _____

1. (6 points) For what values of h and k is the system below is **inconsistent**?

$$\begin{array}{rcrcrcrcrcl} x_1 & + & hx_2 & = & 2 \\ 4x_1 & + & 8x_2 & = & k \end{array}$$

- A. $h = 2, k = 8$
B. $h = 2, k \neq 8$
C. $h \neq 2, k = 8$
D. $h \neq 2, k \neq 8$
E. The system is always consistent
2. (6 points) For what value of x will \vec{y} be in $\text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \vec{y} = \begin{bmatrix} -4 \\ 2 \\ x \end{bmatrix}$$

- A. 1
B. 2
C. 3
D. 4
E. No such x exists

3. (6 points) If

$$\begin{bmatrix} 4 & -3 & 1 \\ 5 & -2 & a \\ b & 2 & -3 \end{bmatrix} \begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -7 \\ -3 \\ 10 \end{bmatrix}$$

for some a and b , find $C_1 - C_2 + C_3$ such that

$$\begin{bmatrix} -7 \\ -3 \\ 10 \end{bmatrix} = C_1 \begin{bmatrix} 4 \\ 5 \\ b \end{bmatrix} + C_2 \begin{bmatrix} -3 \\ -2 \\ 2 \end{bmatrix} + C_3 \begin{bmatrix} 1 \\ a \\ -3 \end{bmatrix}$$

Hint: you do not need to know a and b to answer this question.

- A. -2
 - B. -1
 - C. $\boxed{0}$
 - D. 1
 - E. 2
4. (6 points) If a matrix A is row equivalent to

$$\begin{bmatrix} 1 & 3 & 0 & -4 \\ 4 & 12 & 0 & -16 \end{bmatrix}$$

How many **basic variables** does the homogeneous system $A\vec{x} = \vec{0}$ have?

- A. 0
- B. $\boxed{1}$
- C. 2
- D. 3
- E. 4

5. (6 points) If $A = \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix}$, find the determinant of $3I - A^T$, where I is the identity matrix.

- A. -12
- B. -10
- C. -2
- D. 0
- E. 3

6. (6 points) Which of the following are subspaces of \mathbb{R}^n ?

- I. All vectors (x, y) such that $x \geq 0, y \leq 0$
- II. $\text{Span}\{\vec{v}_1, \vec{v}_2\}$ where $\vec{v}_1 \neq \vec{0}$ and \vec{v}_2 is a multiple of \vec{v}_1
- III. All vectors (x, y) such that $x + y = 1$

- A. I only
- B. II only
- C. III only
- D. I and II only
- E. None of them

7. (6 points) Find all values of a in the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ a & a & 2 \\ 0 & a & 1 \end{bmatrix}$ such that $A\vec{x} = \vec{0}$ has nontrivial solutions.

- A. $a = 1$ only
- B. $a \neq 0$ and $a \neq 1$
- C. $a \neq 1$ only
- D. $a = 0$ only
- E. $a = 0$ and $a = 1$

8. (6 points) Find all values of a and b so that

$$\begin{bmatrix} 1 & 2 \\ a & 0 \end{bmatrix} \begin{bmatrix} 3 & b \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ 12 & 16 \end{bmatrix}$$

9. (10 points) If it exists, find the inverse of the matrix

$$\begin{bmatrix} 1 & -2 & -1 \\ -1 & 5 & 6 \\ 5 & -4 & 5 \end{bmatrix}$$

10. (14 points) If

$$A = \begin{bmatrix} -9 & -6 & 0 & 3 \\ 0 & 6 & -18 & 5 \\ 18 & 9 & 9 & -2 \end{bmatrix}$$

a. Find the null space of A

b. Is $\begin{bmatrix} 3 \\ 5 \\ -2 \\ 0 \end{bmatrix}$ in the null space? Justify your answer. (Hint: you do not necessarily need part a to answer part b.)

11. (10 points) Do the following vectors form a basis of \mathbb{R}^3 ? Why or why not?

$$\begin{bmatrix} 0 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 3 \end{bmatrix}$$

12. Circle at most one response for each question. (2 pts each if correct, -1 if incorrect, 0 if no response)

(a). Two matrices are row equivalent if they have the same number of rows.

TRUE

☐ FALSE

(b). If the augmented matrices of two linear systems are row equivalent, then the systems have exactly the same solution set.

☐ TRUE

FALSE

(c). A free variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix.

TRUE

☐ FALSE

(d). The solution set of the linear system whose augmented matrix is $\begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 & \vec{b} \end{bmatrix}$ is the same as the solution set of the equation $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{b}$.

☐ TRUE

FALSE

(e). The equation $A\vec{x} = \vec{b}$ will always have at least one solution for any \vec{b} if $A\vec{x} = \vec{0}$ has infinitely-many solutions.

TRUE

☐ FALSE

(f). If a set of vectors is linearly dependent, then every vector in the set can be written as a linear combination of the other vectors in the set.

TRUE

☐ FALSE

(g). An 8×5 matrix defines a mapping from \mathbb{R}^5 to \mathbb{R}^8 .

☐ TRUE

FALSE

(h). A mapping $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is one-to-one if each vector in \mathbb{R}^n maps onto a unique vector in \mathbb{R}^m .

TRUE

☐ FALSE

(i). Assuming AB and BA are defined for matrices A and B , then AB is never equal to BA .

TRUE

☐ FALSE