Exam 2

~~2.6~~ - 9.3
9.1

10 questions, 10 points each, same format as last time

one 4×6 note card

---

periodic: $f(t+T) = f(t)$ T: period

e.g. $\sin(t+2\pi) = \sin(t)$

multiples of T is also a period

$\sin(t+6\pi) = \sin(t)$

Fourier series period $2L$

$$g(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + b_n \sin\left(\frac{n\pi t}{L}\right)$$

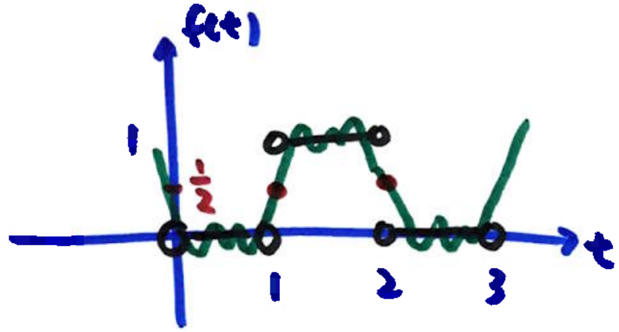
$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi t}{L}\right)dt$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\left(\frac{n\pi t}{L}\right)dt$$

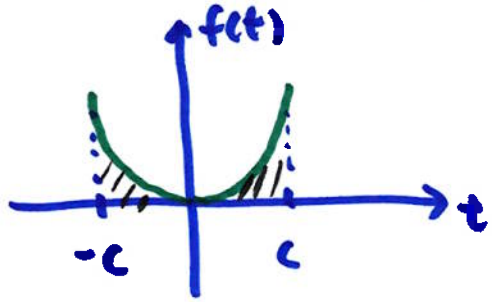
given on exam

$g(t)$ converges to $f(t)$ wherever $f(t)$ is continuous

$g(t)$ converges to $\frac{1}{2}\left[f(t^-) + f(t^+)\right]$ where $f(t)$ is discontinuous

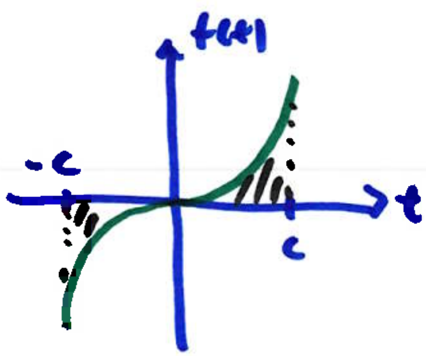


if $f(t)$ is even, $f(-t) = f(t) \rightarrow$ vertical axis symmetry



$$\int_0^c f(t)\,dt = \int_{-c}^0 f(t)\,dt$$

if $f(t)$ is odd, $f(-t) = -f(t) \rightarrow$ origin symmetry



$$\int_0^c f(t)\,dt = -\int_{-c}^0 f(t)\,dt$$

$f(t)$ even: Fourier Cosine series $b_n = 0$ $a_n = \frac{2}{L}\int_0^L f(t)\cos\left(\frac{n\pi t}{L}\right)dt$

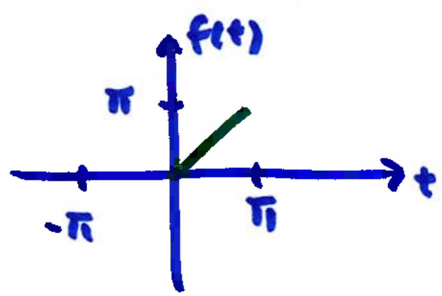
$f(t)$ odd: Fourier Sine Series $a_n = 0$ $b_n = \frac{2}{L}\int_0^L f(t)\sin\left(\frac{n\pi t}{L}\right)dt$

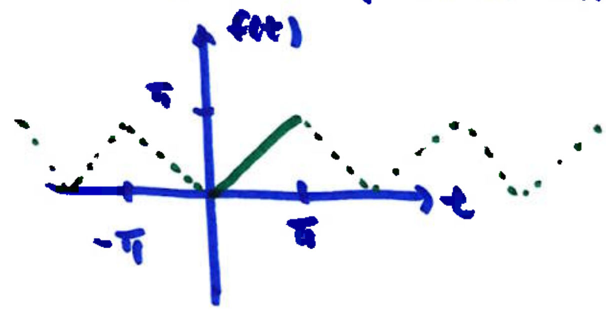
not every function is even or odd

e.g. $f(t) = \cos(t) + \sin(t)$ is neither

even and odd extensions : period $2L$ but given half of it

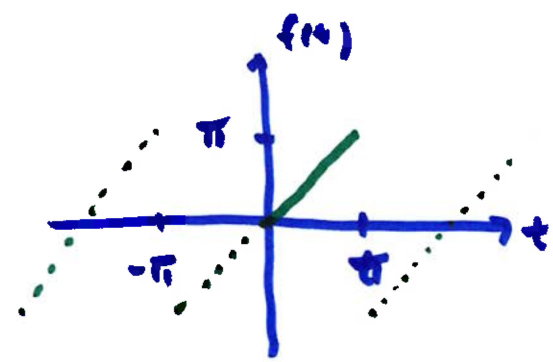
for example, $f(t) = t$ $0 < t < \pi$ period 2π





even

Cosine only



odd

Sine only

boundary-value problems

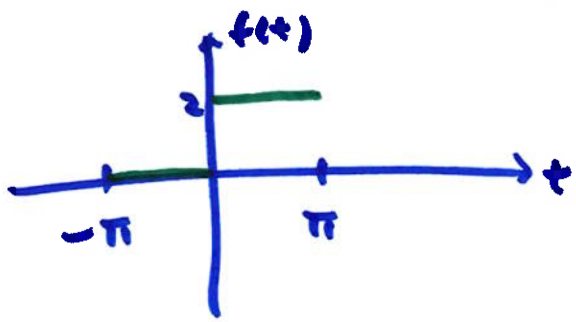
e.g. $x'' + x = f(t)$ $\quad$ $f(t)$ periodic $\quad$ $x(0) = x(L) = 0$

or $x'(0) = x'(L) = 0$

expand $f(t)$ in cosine or sine series to meet boundary conditions

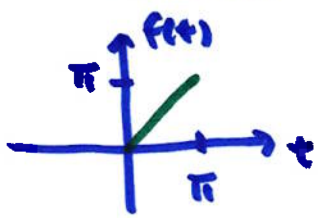
then expand x in Fourier series, sub into ODE, find coefficients

---

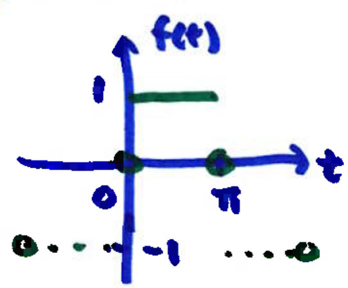


neither even nor odd

$x'' + 2x = t$ $\quad$ $x'(0) = x'(\pi) = 0$ $\quad$ (Quiz 8)



derivative :



→ expand in cosine or sine to meet BC's → sine series in derivative

so if the deriv. is sine

we want to expand $f(t)$ in cosine series

expand t in cosine series, $L = \pi$

$b_n = 0, \quad a_n = \frac{2}{\pi}\int_0^{\pi} f(t)\cos\left(\frac{n\pi t}{\pi}\right)dt = \frac{2}{\pi}\int_0^{\pi} t\cos(nt)\,dt$

$u = t \qquad dv = \cos(nt)\,dt$

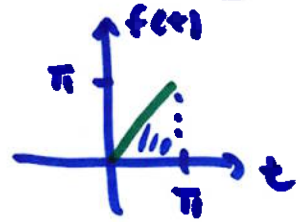
$du = dt \qquad v = \frac{1}{n}\sin(nt)$

$$\frac{2}{\pi}\left[\frac{t}{n}\sin(nt)\Big|_0^{\pi} - \frac{1}{n}\int_0^{\pi}\sin(nt)\,dt\right]$$

$$= \frac{2}{\pi}\left[\frac{t}{n}\sin(nt)\Big|_0^{\pi} + \frac{1}{n^2}\cos(nt)\Big|_0^{\pi}\right] = \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} t\,dt$$

$$= \frac{2}{\pi}\left(\frac{1}{2}\cdot\pi\cdot\pi\right) = \pi$$



$x'' + 2x = t \qquad t \sim \frac{1}{2}\pi + \sum_{n=1}^{\infty}\frac{2}{n^2\pi}\left[(-1)^n - 1\right]\cos(nt)$

expand x: $\quad x = \frac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n\cos(nt) + B_n\sin(nt)$

$$x' = \sum_{n=1}^{\infty} -nA_n\sin(nt) + nB_n\cos(nt)$$

$$x'' = \sum_{n=1}^{\infty} -n^2A_n\cos(nt) - n^2B_n\sin(nt)$$

$$\sum_{n=1}^{\infty} -n^2 A_n \cos(nt) - n^2 B_n \sin(nt) + A_0 + \sum_{n=1}^{\infty} 2A_n \cos(nt) + 2B_n \sin(nt)$$

$$= \frac{1}{2}\pi + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left[(-1)^n - 1\right]\cos(nt)$$

so, $A_0 = \frac{1}{2}\pi$, $B_n = 0$, $A_n = \frac{2\left[(-1)^n - 1\right]}{n^2\pi\,(2-n^2)}$ ~~cos(nt)~~

$$X = \frac{1}{4}\pi + \sum_{n=1}^{\infty} \frac{2\left[(-1)^n - 1\right]}{n^2\pi\,(2-n^2)}\cos(nt)$$