$\int \frac{1}{(1+x^2)} = \arctan(x) + C$

MA262 exam 1 review

- differential equation → $\frac{dy}{dx} = f(x)$ OR $y' = f(x)$ (SLOPE!)
- Solution → solve for y
  - if given initial condition, plug into general solution and solve for C

SOLVING 1st ORDER DIFF EQ:

- right hand side contains no y — $\frac{dy}{dx} = f(x)$ JUST CALC
  - just integrate and solve for y
- right hand side contains y — 2 ways to solve:

1) Separable equation

use only x or ÷

- separate x and y onto either side and integrate both sides
  - ↳ C only has to be on one side
- solve for y for explicit solution, C for implicit

APPLICATIONS

exponential growth / decay → $\frac{dy}{dt} = Ky$

Newton's Law of Cooling → $\frac{dT}{dt} = K(M-T) > \frac{1}{M-T}dT = Kdt$

terminal velocity → $\frac{dv}{dt} = g - \frac{c}{m}v$

$\frac{1}{g - \frac{c}{m}v} dv = dt$ ⟹ also linear

2) Linear equation

1st order standard form: $\frac{dy}{dx} + P(x)y = Q(x)$

manipulate to get this form

integrating factor: $I = e^{\int P(x)dx}$

- multiply I on both sides to get:

$Iy = \int Q(x) I$ → integrate right side, solve for y

↳ DON'T FORGET +C

SLOPE FIELDS

- $\frac{dy}{dx}$ dep. on x → columns are same
- $\frac{dy}{dx}$ dep. on y → rows are same
- $\frac{dy}{dx}$ dep. on x AND y:
  - identify where $\frac{dy}{dx} = 0$
  - consider how slope will behave above and below zero slope line

Linear equations APPLICATIONS - mixing problem

$\frac{dy}{dt} = r_i C_i - r_o C_o$

$= r_i C_i - r_o \frac{y}{V(t)}$

$C_o = \frac{y(t)}{V(t)}$ → amount of salt / → solution volume

- if r_i is diff from r_o: $V(t) = \text{initial} + t$

- manipulate into linear and follow same procedure

SUBSTITUTION METHODS:

- **Linear substitution**

form: $\frac{dy}{dx} = f(ax+by+c)^n$

- turns into SEPARABLE equation → solve for y

sub: $v = ax + by + c$

$v' = a x \frac{d}{dx} + by \frac{dy}{dx} + c$

- **Homogeneous substitution**

form: can be written as $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

sub: $v = \frac{y}{x}$ → $y = vx$

$y' = v'x + v$

$v'x + v = f(v)$ move v term to right: $\frac{dv}{dx} x = f(v) - v$

- get SEPARABLE equation: $\frac{1}{f(v)-v} dv = \frac{1}{x} dx$
- integrate and solve for v
- replace v with $\frac{y}{x}$
- solve for y if possible

- **Bernoulli's Equation**

Form: $y' + P(x)y = G(x)y^n$

sub: $v = y^{1-n}$

$v' = (1-n)y^{-n} \cdot y'$

rearrange into standard form

- divide both sides by y^n: $\underbrace{y^{-n}y'}_{\text{almost } v'} + P(x)\underbrace{y^{1-n}}_{v} = Q(x)$
- make sub: $\frac{1}{1-n} v' + P(x)v = Q(x)$
- solve as LINEAR equation → solve for v
- plug back $v = y^{1-n}$ solve implicitly / explicitly y

EXACT EQUATIONS

$df = \underbrace{\frac{\partial f}{\partial x}}_{M} dx + \underbrace{\frac{\partial f}{\partial y}}_{N} dy = 0$

$f_x\, dx + f_y\, dy = 0$

- test if equation is exact:

$M_y = N_x$

- if so, implicit solution exists

- pick M or N to integrate and use other to find remaining constant to solve $f(x)$

2 SPECIAL KINDS OF 2nd ORDER DIFF EQ TO USE SUB TO SOLVE AS 1st ORDER DIFF EQ:

1) no x in equation

sub: $p = y'$, $p' = y'' = \frac{dp}{dy} \cdot \frac{dy}{dx}$

- get SEPARABLE → solve fo p
- sub $p = \frac{dy}{dx}$ and get another SEPARABLE → solve for y

2) no y in equation

sub $p = \frac{dy}{dx} = y'$