Exam 1 261 Exam

13.1 - 13.4

- Magnitude: $|\vec{u}| = \sqrt{a^2+b^2}$
- Unit Vector: $\frac{\vec{u}}{|\vec{u}|}$
- Dot Product: $\vec{u}\cdot\vec{v} = |u||v|\cos\theta$
  - If $u \cdot v = 0$ vectors $\perp$
- Cross product: $|u \times v| = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = i \quad -j \quad +k$
  - If $u \times v = 0$ vectors $//$
- Triangle area $= \frac{1}{2}|u \times v|$
- Circle: $(x-h)^2 + (y-k)^2 = r^2$
- Projection: $\text{Proj}_a b = \frac{a \cdot b}{|a|} \cdot \frac{a}{|a|}$
  (b onto a)

13.5

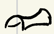
- Line:
  - Vector form: $\vec{r}(t) = \vec{r_0} + t\vec{v}$
  - Parametric form: $x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

- Plane: $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

- If a line or plane is parallel to the another it means that it's orthagonal to the normal vector

- If dot product of normal vectors $= 0$ then lines/planes $\perp$, if $a \cdot b = |a||b|\cos\theta$, $\cos\theta = 1$ lines/planes parallel

13.6

- Ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ 
- Cone: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$ 
- Hyperbolic Paraboloid: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = z$ 
- Paraboloid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$ 
- Hyperboloid of one sheets: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
- Hyperboloid of two sheets: $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

- When manipulating don't just assume, try to draw out planes for better visual

14.1

- If given $r(t) = \langle t\cos t, t\sin t \rangle$ use $\cos^2 t + \sin^2 t = 1$ and set variables equal to x, y, z to find proper equation

14.2 - 14.3

- Unit tangent vector: $T = \frac{r'(t)}{|r'(t)|}$
- Vector integration:
  1) Integrate and add "C" vector
  2) Set t to whatever variable given
  3) set x, y, z variable of integral equal to point, solve for C_x, C_y, C_z

14.4 - 14.5

- Curve length: $L = \int_a^b |r'(t)|\,dt$
  - If $|r'(t)| = 1$ arc length is parameter
  - If $|r'(t)| \neq 1$ and t is left over
    1) $s(t) = \int_a^t |r'(t)|$ set original t to u
    2) Integrate
    3) Solve for t and plug into original $r(t)$
    4) Re-set bounds: plug old bounds into $s(t)$
- Curvature: $K = \frac{|T'|}{|r'|} = \frac{|r'' \times r'|}{|r'|^3}$

15.1

- Level curve: $z = K$ ⟶ set $f(x,y) = K$ and evaluate
  K = any # therefore try 0, 1, 2, etc. till resulting in recognizable shape
  - Parabola: $y = ax^2 + bx + c$
    $y = x^2$ or $y = x^2 - x$
  - Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

15.2

- $\lim_{u \to 0} \frac{\sin u}{u} = 1$

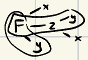
1) Test using given values
2) Test $x=a$ or $y=b$ (any value, 0, 1, 2, etc.) and $x=y$
   - If same value continue, otherwise DNE
3) Manipulate / u-sub

15.3

- f_{xy}: read left to right
- $\frac{\partial^2 f}{\partial y \partial x}$: read right to left

- $\frac{\partial z}{\partial y} \longrightarrow$ End variable, use every path with it

Ex)



$F_y + F_z \cdot z_y$

15.4

- Partial at splits, regular if no split
- Implicit differentiation: Make new function $(F = \ldots = 0)$, tree, the evaluate

15.5

- Directional Derivative: $D = \nabla \cdot u = \underbrace{\langle f_x, f_y \rangle}_{\text{Gradient}} \cdot \underbrace{\langle u, u \rangle}_{\text{unit vector (direction)}}$
- Max increase: $\frac{\nabla}{|\nabla|}$
- Max decrease: $-\frac{\nabla}{|\nabla|}$
- Max change: $|\nabla|$
- No Gradient: Point $\cdot \langle x, y \rangle = 0$
- Slope of tangent line $\frac{dy}{dx} = \frac{-f_x}{f_y}$

15.6

- Tangent Plane: $\vec{\nabla} F \cdot \langle x - x_0, y - y_0, z - z_0 \rangle$
- Linear approximation: $\vec{\nabla} \cdot \langle x - x_0, y - y_0, z - z_0 \rangle + f(x, y)$
- Precent Change: $\frac{dz}{z}$
- Change in z: $dz = f_x dx + \underbrace{f_y dy}_{\text{Evaluate at } (x_0, y_0)}$