15.7 Max and Min problems

- Critical point: $F_x = 0$ and $F_y = 0$

$D(x,y) = f_{xx} f_{yy} - (f_{xy})^2$

Min: $D > 0$ $f > 0$

Max: $D > 0$ $f < 0$

Saddle: $D < 0$

Inconclusive: $D = 0$

15.8 Lagrange Multipliers

$\nabla f = \lambda \nabla g$

1) Break equation into x, y, z components and solve for a commonality between 3
2) Set components equal to each other and solve for singular component, x, y, or z
3) Plug solved variable into $g(x,y,z)$ solve for $g = 0$ then plug into other components to get CP
4) Plug CP into $f(x,y,z)$

OR: Get $\nabla F = \lambda \nabla G$ to equal zero and plug into G for CP: $F_x - \lambda G_x = 0$ and $F_y - \lambda G_y = 0$ (λ value doesn't matter)

16.1 Double Integrals Over rectangular regions

$$\underset{x}{\int_a^b} \underset{y}{\int_c^d} f(x,y)\,dy\,dx$$

$$f_{avg} = \underbrace{\frac{1}{A}}_{\text{Area}} \underset{x\,y}{\iint} f(x,y)\,dy\,dx$$

16.3 Integrals in Polar Coordinates

$x^2 + y^2 = r^2$

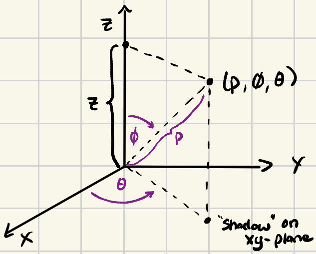
$x = r\cos\theta$

$y = r\sin\theta$

$$\underset{z\ \theta\ r}{\iiint} r\,dr\,d\theta\,dz$$

$dA = r\,dr\,d\theta$

16.5 Triple Integrals in Spherical Coordinates



$\rho \geq \emptyset$

$0 \leq \theta \leq 2\pi$

$0 \leq \emptyset \leq \pi \longrightarrow$ Max distance

$x^2 + y^2 + z^2 = \rho^2$ $\qquad$ $dV = \rho^2 \sin\emptyset \, d\rho \, d\emptyset \, d\theta$

$x = \rho \cos\theta \sin\emptyset$

$y = \rho \sin\theta \sin\emptyset$

$z = \rho \cos\emptyset$

16.6 Integral in Mass Calculations

Moment: $M_x = \iint_R y \, \rho(x,y) \, dA$

$M_y = \iint_R x \, \rho(x,y) \, dA$

mass: $m = \iint_R \rho(x,y) \, dA$ $\quad$ or $\quad$ $m = \iiint_R \rho(x,y,z) \, dV$

x coordinate: $\bar{x} = \frac{1}{m} \iint_R x \, \rho(x,y) \, dA$

y coordinate: $\bar{y} = \frac{1}{m} \iint_R y \, \rho(x,y) \, dA$

z coordinate: $\bar{z} = \frac{1}{m} \iiint_R z \, \rho(x,y,z) \, dV$

17.1 Vector Fields

- Potential function: Scalar function
- Gradient vector field: gradient of some scalar

Potential $\longrightarrow$ Gradient

$G = \nabla P = \langle \partial x, \partial y \rangle$

17.2 Line Integrals of functions and vector fields

- $ds = |r'(t)|\,dt$
- Work $= \int_a^b F \cdot r'\,dt$

- Bounds of line: $r(t) = \langle \text{Starting point} \rangle + t\langle \text{Final - inital point} \rangle$

  $r(t) = \langle a, b \rangle + t\langle c, d \rangle$

  Find values for t that makes $r(t) =$ final and initial point

- Solving for $r(t)$ given points and equations

  Ex) $\int_c (x+\sqrt{y})\,ds$ from $(0,0)$ to $(1,1)$ along $y = x^2$ then back to $(0,0)$ along $y = x$

  - If equation is line use line equation and pick direction
  - If equation other than line, pick x to be t and change y to match original equation:

    Ex) If $x = t$

    $y = t^2$

    $r(t) = \langle t, t^2 \rangle$

Reminder Notes

- Cylindrical:

  $r\,dr\,d\theta = dA$

  $r^2 = x^2 + y^2$

  $x = r\cos\theta$

  $y = r\sin\theta$

- Area under a plane:

  $$\iint \sqrt{F_x + F_y + 1}\; dA$$

- Max and Min

  $\nabla F = \lambda \nabla G$

  $D > 0 \quad F_{xx} < 0 \quad$ max

  $D > 0 \quad F_{xx} > 0 \quad$ min

  $D < 0 \quad$ saddle

  $D = 0 \quad$ undetermined

- Spherical:

  $\rho^2 \sin\phi \, d\rho\, d\phi\, d\theta = dV$

  $\rho^2 = x^2 + y^2 + z^2$

  $z = \rho\cos\phi$

  $x = \rho\sin\phi\cos\theta$

  $y = \rho\sin\phi\sin\theta$

- Work: $W = \int_a^b F \cdot r' \, dt$
- Line integral: $\int f(x,y)\, ds \qquad ds = |r'(t)|\, dt$

- Mass: $m = \iint \text{density}(x,y)\, dx\, dy$

  $\bar{x} = \frac{1}{m} \iint x \text{ dens}(x,y)\, dx\, dy$

  $\bar{y} = \frac{1}{m} \iint y \text{ dens}(x,y)\, dx\, dy$

  Moment: $X = \iint y \text{ dens}$

  Moment of inertia: $I = \iint \rho d^2$ d: distance from axis being rotated about