Mathematics and the Mathematical Sciences in 2010: What Should Students Know?

ABSTRACT: This report is a call for participation from the mathematics profession. CUPM is gathering information that will conclude with the preparation of a Curriculum Guide that will serve as a useful tool for departments as they consider their undergraduate curriculum. CUPM has held paper sessions, panels, and focus groups. Faculty from mathematics and mathematically intensive disciplines have met together to discuss the mathematical needs of other disciplines. In addition, as part of this information gathering, CUPM invited a number of mathematicians to write thoughtful and provocative papers describing their own views on a variety of topics of interest to CUPM. To summarize this activity, CUPM met with the authors at a small three-day workshop to isolate major issues and develop preliminary and tentative recommendations for a Curriculum guide to report to the profession for comment and discussion. CUPM is listening to you.

What are your views on the mathematics curriculum? Faculty expend great effort thinking about, preparing for, and facilitating the learning of mathematics. Every child is now promised a college education. The job market values analytical skills above all others. Students graduating in mathematics take jobs and pursue further education in an ever-widening number of areas. Students who, in the past, might have majored in mathematics now are concentrating in allied mathematically intensive disciplines. A new surge of students is entering higher education expecting faculty to prepare them in their discipline and provide the background for future careers. Acute teacher shortages are developing in mathematics and science throughout the United States. Universities are instituting procedures of accountability. What is the appropriate program in this new environment? The community of mathematicians, while working with experts in other disciplines, is best able to respond.

Please, page through this report; think about the possible recommendations; read in depth the ideas of the articles that ring true for you; analyze those questions that bother you; but above all, discuss curricula and expectations with your colleagues and report your views to CUPM (CUPM-curric@maa.org).

I. The Goal of the CUPM Curriculum Initiative

The Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America (MAA) issues curriculum guidelines about once every decade. The present committee is gathering information leading to the curriculum guide. With far greater diversity among institutions, preparations of students, and expectations of higher education, it is no longer possible to just list and describe the content of a few courses that would constitute a major. We must ask ourselves, "What should students know?" The process of answering this question will then serve as a useful process on which to build a curriculum addressing the needs of a particular college or university. With this in mind, CUPM will prepare a document differing from those in the past. The focus of the Guide will not be "What courses should a department offer or require?" but rather "What do we want our students to know and be able to do?" Planning a program in answer to this question is a process. A good deal of thought, information, and discussion are required. CUPM hopes to provide guidance to departments as they go through this process.

To facilitate a planning process, CUPM can assist departments by posing the right questions, suggesting a structure to find answers, and providing examples and information so that departments can benefit from the experience of colleagues elsewhere. The preliminary recommendations are organized into seven broad areas outlined below.

A. Mathematical Knowledge

What should students know upon graduation? Student knowledge grows in several ways. Closest to our own discipline are recommendations about the mathematical content knowledge that we expect students to understand. But as a second critical foundation, we also expect a level of skill development. A new third dimension has been added as technology has grown in importance and as more of our students enter the job market directly. Students now may need experience with powerful mathematical software tools not only to promote learning in mathematics, but also to be prepared for a wide range of software tools they will encounter later.

B. Experience Supporting Knowledge

Should every student be given an opportunity to "put it all together" through a group or individual experience? Employers expect graduates to be able to read, write, and speak about mathematics both for their expert colleagues and for others whose background is not in mathematics. We expect our graduates to enjoy mathematics and to be emissaries for it in later life.

C. A Special Responsibility to Future Teachers

How often have you met someone casually, explained that you "did mathematics," and heard in reply, "Math, I could never do that?" Our first lines of defense against such illiteracy are the teachers in our schools. Almost as never before, the country is encountering critical shortages of mathematics and science teachers. We not only want quality teachers who love mathematics, but we want those teachers to teach as we want our children to learn. Many universities assume primary responsibility for preparing future teachers. But other universities also have a number of mathematics students who contemplate becoming teachers. Every department should discuss the role of their program in the preparation of future teachers of mathematics and science.

D. The Environment Outside Class

Some of the most successful departments have thought about the out-of-class experience for their students. Because motivation so strongly influences learning, these departments have fostered a large number of loyal students. What parts of the environment should a department address? What actions benefit students?

E. Assessment

We would all like to know that we are succeeding. Such knowledge comes from knowing our goals and gathering information that would tell us we are reaching them. How are our students being assessed? Is this compatible with reaching our expectations of them? Is our program succeeding? Are we recruiting, retaining, and graduating the audience we expected? Are students reporting their satisfaction with their preparation once they leave us?

F. The Department and Its Capacity

Consider

(a) a major land grant university serving 20,000 students in a region that includes an entire state,
(b) a state university of 13,000 students chartered to serve everyone in a local region,
(c) a privately endowed university charged to reach out to the "best and the brightest" throughout the world,
(d) a college of 2,000 students focusing on undergraduate education in the liberal arts drawn from across the country, and
(e) a college of 1,500 undergraduates serving particular needs in a local area.

These institutions will serve their students in many different kinds of classroom settings providing a wide range of opportunities. Clearly, the programs at these differing institutions will be different, in courses, in sequence, and in experiences. A program designed locally will best serve the students in a particular institution. But doesn't it seem reasonable that students graduating with intensive mathematical training from any of these institutions should possess knowledge and skills that are recognizable as preparation in mathematics?

If the faculty is too assist students to reach our expectations, then what are the needs of the faculty? How should expectations be set and fulfilled so that faculty members enjoy their careers and grow as teachers and mathematicians?

G. Issues for the Discipline

The Curriculum Guide will be the first of its type from CUPM. As such there is a responsibility to enhance the usability in the future and to provide departments with the kinds of information they need. Models That Work (MAA Notes XX) provided examples of successful programs. Toward Excellence (AMS ???) provided a more focused look at research universities. What should the profession provide to departments?

Between 4 and 5 percent of an age group majors in Mathematics, Science, or Engineering, that is, one of the traditional mathematics intensive disciplines. This percentage has been fairly constant since the 1950's through both mathematics reform and "back-to-basics" movements. Majors grow or shrink by reapportioning students in this 5 percent group. In this decade, our mathematics students are frequently in other disciplines. Is there a responsibility to reach out to them? What can the profession do to assist departments as they address the issues?

II. The Information Gathering Process

Prior to this Report, CUPM has engaged in a series of information gathering projects. First, this curriculum report builds upon past CUPM recommendations. In 1981 CUPM recognized that many students wish to combine their mathematics with knowledge from other disciplines in order to broaden their knowledge and enhance their future career opportunities. This curriculum was reissued in 1988 as Reshaping College Mathematics (MAA Notes 13). The 1991 CUPM recommendations appear at the end of Heeding the Call for Change (MAA Notes 22). By this time, a list of courses for the major was no longer easy to state and the mathematics community was engaged in major discussions about calculus. Therefore, the curriculum document is brief. Since 1991, Models That Work (MAA Notes 38) described exemplary programs in mathematics, Confronting the Core Curriculum (MAA Notes ??) addressed preparation in the first two years, and, most recently, (The latest book on assessment MAA Notes ??) addressed a broad range of assessment issues with models illustrating the issues. All of these documents remain useful for departments planning their programs. They also serve as a standard from which we are evolving.

More recently, CUPM has been gathering information directly from the profession in the following ways:

A. Meeting sessions and panels.
   At Mathfest 99, CUPM sponsored a panel/audience reaction session in front of an involved and packed audience.
   At the January 00 AMS/MAA/SIAM meeting CUPM sponsored a panel/audience reaction session again to a packed room, plus two very well attended contributed paper sessions.
   At Mathfest 00 CUPM sponsored a panel of industry members commenting on the curriculum for a full and active audience.

B. Focus Groups
   In January 00 CUPM invited mathematicians to participate in a number of focus groups discussing curricular issues by institution type.

C. Interdisciplinary Conferences
   The subcommittee Calculus Reform and the First Two Years (CRAFTY) has held a sequence 11 different workshops bringing mathematicians together with faculty from other mathematics intensive disciplines to discuss the undergraduate curriculum. From these meetings a sequence of reports will be issued by those other disciplines on the mathematical needs of their students. The final workshops will be held in 2001 and the reports from the various disciplines will be combined for distribution later that year. The reports themselves will be made available prior to that time on the web.

D. Solicited Papers followed by a Summative Workshop
   The solicited papers appear in the Report along with the deliberations and tentative recommendations that CUPM would like to discuss with the larger community.

E. A Call for Discussion

This Report will be issued in time for the January 01 AMS/MAA meetings. At that meeting a general and extended panel session will elicit comment from the community. Further, another sequence of invited focus groups will be held to obtain deeper reaction from mathematicians.

F. Involvement of Mathematics Intensive Disciplines

Representatives of sister societies in engineering, physics, and economics attended the discussion reported on here. AMS has representation on CUPM through membership on CUPM and through a liaison appointed by the President of AMS. The presidents of MAA, SIAM, and AMATYC contributed papers and participated in the workshop. During the next year, liaisons from a broad range of additional disciplines will be asked to contribute to CUPM's deliberations.

CUPM seeks to involve the major stakeholders with greatest emphasis on the mathematics profession itself. If we can engage you in active discussion, with your colleagues and through your profession then we will have already done a great deal for curricular awareness and improvement. Further, through the information you will provide us, we will be able to offer the best guidance and advice.

We have had regular articles appear in Focus and there will be more. We have an email address that directly reaches CUPM and open for your use (CUPM-curric@maa.org). By the time this appears we hope also to have a web page. CUPM is listening to you.