21 $F = x^2 + 2y^2 + 3z^2$

$$\Rightarrow \nabla F = (2x,\ 4y,\ 6z)\Big|_{(1,1,-1)} = (2,4,-6) \parallel (1,2,-3)$$

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} x-1 \\ y-1 \\ z+1 \end{bmatrix} = (x-1) + 2(y-1) - 3(z+1) = 0$$

$$\Longleftrightarrow \quad x + 2y - 3z = 6$$

---

22. $f(x,y) = \pi + \sin(\pi x^2 + 2y)$, $\quad f(x,y)\ f(2,\pi) = \pi$

$$f_x = \cos(\pi x^2 + 2y) \cdot 2\pi x \Big|_{(2,\pi)} = 4\pi$$

$$f_y = \cos(\pi x^2 + 2y) \cdot 2 \Big|_{(2,\pi)} = 2$$

$$z - \pi = 4\pi(x-2) + 2(y-\pi)$$

$$\Longleftrightarrow \quad 4\pi x + 2y - z = 9\pi$$

---

23. $f(x,y,z) = x e^{y^2 - z^2}$

$$df = e^{y^2-z^2}\, dx + 2xy e^{y^2-z^2} - 2xz e^{y^2-z^2}$$

$f_x = e^{y^2-z^2}$

$f_y = 2yx\, e^{y^2-z^2}$

$f_z = -2xz\, e^{y^2-z^2}$

---

24. $f(x,y) = 2x^3 - 6xy - 3y^2$

$f_x = 6x^2 - 6y = 0$

$f_y = -6x - 6y = 0 \quad \Rightarrow x = -y$

$6x^2 + 6x = 0 \Leftrightarrow 6x(x+1) = 0 \quad x = 0, -1$

$f_{xy} = -6$

$f_{xx} = 12x$

$f_{yy} = -6$

$D = -72x - 36$

@ $x = 0$ $D < 0$, $\Rightarrow$ saddle

@ $x = -1$ $D > 0$, $f_{xx} < 0 \Rightarrow$ max

25 $f(x,y) = 4x^2 + y^2$ $\qquad g(x,y) = xy = 1$

$$\nabla f = \begin{bmatrix}8x\\2y\end{bmatrix} = \begin{bmatrix}\lambda y\\ \lambda x\end{bmatrix} = \lambda\nabla g$$

$$y\,8x = \lambda y^2, \qquad x\,2y = \lambda x^2$$

$$\cancel{\Rightarrow \frac{8x}{y} = \lambda \Rightarrow 2y = \frac{8x}{y}\,x \Rightarrow y^2 = 4x^2}$$

$$8xy = 8 = \lambda y^2, \qquad 2xy = 2 = \lambda x^2$$

$$\frac{8}{y^2} = \frac{2}{x^2} \qquad 8x^2 = 2\cancel{y^2}\,\frac{1}{x^2}$$

$$x^4 = \frac{1}{4} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

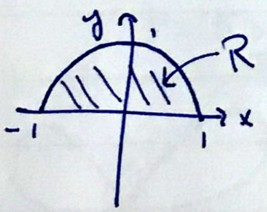
$$\cancel{\tfrac{1}{\sqrt2}}\,2\sqrt{2} = \lambda\left(\frac{1}{\sqrt{2}}\right)^2$$

$$4\sqrt{2} = \boxed{\lambda = 4}$$

$$\boxed{8x = \lambda y \Rightarrow \lambda = 4}$$

26. $$\int_1^3\int_0^x \frac{1}{x}\,dy\,dx$$

$$= \int_1^3 \left[\frac{y}{x}\right]_{y=0}^{y=x} dx = \int_1^3 (1-0)\,dx = (3-1) = 2.$$

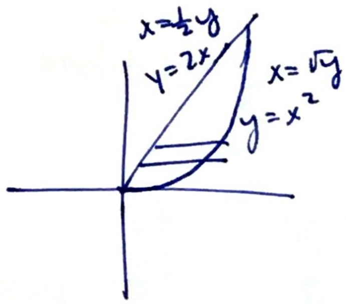
27. 

$$\int_{-1}^{1}\int_0^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$$

$$= \int_0^{\pi}\int_0^1 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

28

$$\int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx = \int_0^4 \int_a^b f(x,y)\,dx\,dy$$

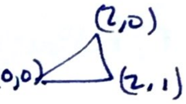


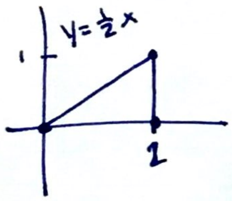
$a = \frac{1}{2}y, \quad b = \sqrt{y}$

29.

$$\iint_R y\,dA$$

R





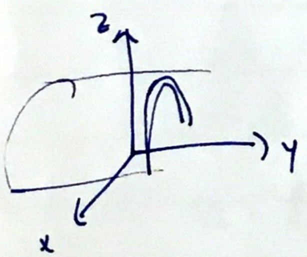
$$= \int_0^1 \int_0^{\frac{x}{2}} y\,dy\,dx$$

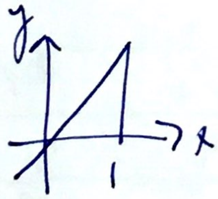
$$= \int_0^2 \left[\frac{1}{2}y^2\right]_{y=0}^{y=\frac{x}{2}} dx = \int_0^1 \frac{1}{8}x^2\,dx$$

$$= \frac{1}{24}x^3 \Big|_0^2 = \frac{8}{24} = \frac{1}{3}$$

---

30. bdd above - $z = \cancel{x}\ 1-x^2$, below - xy-plane, sides $y=0, y=x$

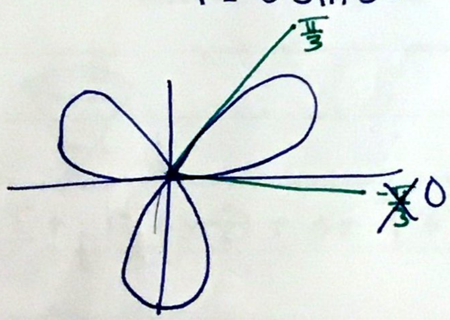




$$\int_0^1 \int_0^x (1-x^2)\,dy\,dx$$

---

31 $r = 5\sin 3\theta$



$D = \{(r,\theta) \mid -\frac{\pi}{3} \le \theta \le \frac{\pi}{3},\ 0 \le r \le 5\sin 3\theta\}$

$$\int_{\cancel{-\frac{\pi}{3}}\ 0}^{\frac{\pi}{3}} \int_0^{5\sin 3\theta} r\,dr\,d\theta$$

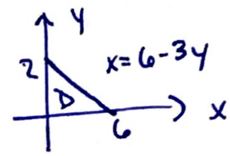
$$= \int_{\cancel{-\pi/3}\ 0}^{\pi/3} \frac{(5\sin 3\theta)^2}{2}\,d\theta = \frac{25}{2}\cdot\frac{1}{2}\int_{\cancel{-\frac{\pi}{3}}\ 0}^{\frac{\pi}{3}} \left[1-\cos(6\theta)\right]d\theta$$

$$= \frac{25}{4}\left[\theta - \frac{1}{6}\sin(6\theta)\right]_{\cancel{-\frac{\pi}{3}}\ 0}^{\frac{\pi}{3}} = \frac{25}{4}\left[\left(\frac{\pi}{3}-0\right) - \left(\cancel{-\frac{\pi}{3}}\ 0 - 0\right)\right]$$

$$= \boxed{\frac{25\pi}{12}}$$

32 $x+3y+2z=6$ in first oct

$x+3y=6$



$SA = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dA$

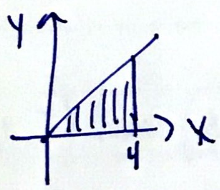
$z = 3 - \frac{1}{2}x - \frac{3}{2}y$

$= \int_0^2 \int_0^{6-3y} \sqrt{1+\frac{1}{4}+\frac{9}{4}}\, dx\, dy = \frac{\sqrt{14}}{2}\int_0^2 (6-3y)\, dy$

$= \frac{\sqrt{14}}{2}\left(6y - \frac{3}{2}y^2\right)\Big|_0^2 = \frac{\sqrt{14}}{2}\left(12 - \frac{3}{2}\cdot 4\right) = \frac{\sqrt{14}}{2}\cdot 6 = \boxed{3\sqrt{14}}$

---

33. $z=y^2,\ y=x,\ y=0,\ z=0,\ x=4$



$E = \{(x,y,z) \mid 0 \le x \le 4,\ 0 \le y \le x,\ 0 \le z \le y^2\}$

$\int_0^4 \int_0^x \int_0^{y^2} 1\, dz\, dy\, dx$

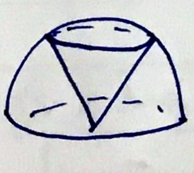
$= \int_0^4 \int_0^x y^2\, dy\, dx = \int_0^4 \left[\frac{1}{3}y^3\right]_{y=0}^{y=x} dx = \int_0^4 \frac{1}{3}x^3\, dx$

$= \frac{1}{12}x^4\Big|_0^4 = \frac{4^4}{3\cdot 4} = \frac{4^3}{3} = \boxed{\frac{64}{3}}$

---

34 Above: $x^2+y^2+z^2=32$, Below: $z^2 = x^2+y^2$

$\rho = \sqrt{x^2+y^2+z^2} = z$



$E = \{(x,y,z) \mid \sqrt{x^2+y^2} \le z \le \sqrt{32-x^2-y^2},$

$-\sqrt{16-x^2} \le y \le \sqrt{16-x^2}$

$-4 \le x \le 4\}$

$2z^2 = 32$

$z^2 = 16$

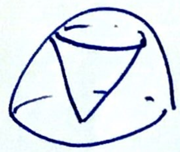
$\quad \shortparallel$

x^2+y^2

$x^2+y^2+z^2 = \rho^2 = 32$

$z^2 = x^2+y^2$

35



$z = \sqrt{x^2+y^2} = \frac{\pi}{4}$ $\qquad \rho = z = \rho\cos\varphi$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sqrt{32}} \rho^3 \cos\varphi \sin\varphi \, d\rho \, d\varphi \, d\theta$$