MA 261

Quiz 1 19 January 2016

**Instructions:** Write your name and section number on your quiz. Show all work with clear logical steps. No work or hard-to-follow work will lose points.

**Problem 1.** (5 points)Find an equation of the plane that passes through the point $(1, 2, 3)$ and contains the line $x = 3t, y = 1 + t, z = 2 - t$.

*Solution.* Since the line lies in the plane, the vector $\mathbf{p} = \langle 3, 1, -1 \rangle$ lies in the plane. It's easy to see that the point $(1, 2, 3)$ is not on the line. So the vector $\mathbf{q}$ from a point on the line to $(1, 2, 3)$ will be nonparallel to $\mathbf{p}$, and thus $\mathbf{p}, \mathbf{q}$ will span the plane. Substituting $t = 0$, we find $(0, 1, 2)$ is on the line, so that $\mathbf{q} = \langle 1 - 0, 2 - 1, 3 - 2 \rangle = \langle 1, 1, 1 \rangle$ . Now

$$\mathbf{n} = \mathbf{p} \times \mathbf{q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \langle 2, -4, 2 \rangle$$

is normal to the plane, and an equation of the plane is

$$\begin{aligned} &2(x - 1) - 4(y - 2) + 2(z - 3) = 0 \qquad \text{or} \\ &2x - 4y + 2z = 0. \end{aligned}$$

☺

**Problem 2.** (5 points)Find parametric equations for the line segment from $(10, 3, 1)$ to $(5, 6, -3)$.

*Solution.* We know that the equation of the segment from $\mathbf{r_0}$ to $\mathbf{r_1}$ is

$$\mathbf{r} = (1 - t)\mathbf{r_0} + t\mathbf{r_1}, \qquad 0 \leq t \leq 1.$$

Here, $\mathbf{r_0} = \langle 10, 3, 1 \rangle$ and $\mathbf{r_1} = \langle 5, 6, -3 \rangle$ . So,

$$\begin{aligned} \mathbf{r} &= (1 - t) \langle 10, 3, 1 \rangle + t \langle 5, 6, -3 \rangle \\ &= \langle 10, 3, 1 \rangle + t \langle -5, 3, -3 \rangle, \qquad 0 \leq t \leq 1. \end{aligned}$$

So the parametric equations are given by

$$\begin{cases} x = 10 - 5t \\ y = 3 + 3t \\ z = 1 - 4t. \end{cases}$$ ☺

**Problem 3.** (5 points)Find the component of $\mathbf{p}$ onto $\mathbf{q}$, where $\mathbf{p} = \langle -2, 3, 6 \rangle$ and $\mathbf{q} = \langle 5, -1, 4 \rangle$.

*Solution.* $|\mathbf{q}| = \sqrt{25 + 1 + 16} = \sqrt{42}$. So the component of $\mathbf{p}$ onto $\mathbf{q}$ is

$$\text{comp}_{\mathbf{q}}\, \mathbf{p} = \frac{\mathbf{p} \cdot \mathbf{q}}{|\mathbf{q}|} = \frac{11}{\sqrt{42}}$$ ☺

**Problem 4.** (0 points) What's your favorite ice cream flavor?

*Solution.* Homemade strawberry. ☺