# MA 261

Quiz 2 | 26 enero 2016

**Instructions:** Write your name and section number on your quiz. Show all work, with clear, logical steps. No work or hard-to-follow work will lose points.

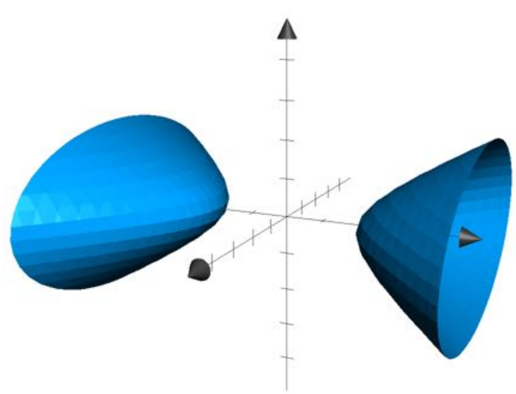
**Problem 1.** (3 points) [From Stewart: 12.6.32] Classify the surface and sketch:

$$y^2 = x^2 + 4z^2 + 4$$

*Solution.*

$$\begin{aligned} y^2 &= x^2 + 4z^2 + 4 \\ &\Leftrightarrow -x^2 + y^2 - 4z^2 = 1 \\ &\Leftrightarrow \frac{-x^2}{4} + \frac{y^2}{4} - z^2 = 1 \end{aligned}$$

is a hyperboloid of two sheets along the y-axis.



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**Problem 2.** (4 points)[From Stewart: 13.1.2] Find the domain of the vector-valued function

$$\mathbf{r}(t) = \left\langle \frac{t-2}{t+1}, \sin t, \log(9-t^2) \right\rangle$$

*Solution.* The domain of $\mathbf{r}$ depends only on the domain of its components. The domain of $\frac{t-2}{t+1}$ is $\mathbb{R} \setminus \{-1\}$. The domain of $\sin t$ is $\mathbb{R}$. log can only take positive arguments, so we need

$$\begin{aligned} 9 - t^2 > 0 &\Leftrightarrow 9 > t^2 \\ &\Leftrightarrow |t| < 3 \\ &\Leftrightarrow -3 < t < 3. \end{aligned}$$

So the domain of $\mathbf{r}$ is $(-3,-1) \cup (-1,3)$. ☺

**Problem 3.** (5 points) [From Stewart: 13.1.48] Two particles travel along the space curves

$$\begin{aligned} \mathbf{r}_1(t) &= \left\langle t, t^2, t^3 \right\rangle \\ \mathbf{r}_2(s) &= \langle 1+2s, 1+6s, 1+14s \rangle . \end{aligned}$$

At what points to their paths intersect?

*Solution.* To find where their paths intersect, we set $\mathbf{r}(t) = \mathbf{r}(s)$. This gives

$$\begin{cases} t = 1 + 2s \\ t^2 = 1 + 6s \\ t^3 = 1 + 14s \end{cases} \tag{1}$$

Note that since there are 3 equations and only 2 variables, we don't need to use the third equation to find values of $t, s$ (though we do need to verify that the solutions we find work). Substituting $t$ in the second equation, we get

$$\begin{aligned} (1+2s)^2 &= 1 + 6s \\ 1 + 2s + 2s^2 &= 1 + 6s \\ 2s^2 - 4s &= 0 \\ 2s(s-2) &= 0 \\ s &= 0, \frac{1}{2} \end{aligned}$$

Now $s = 0 \implies t = 1 + 2(0) = 1$, and $s = \frac{1}{2} \implies t = 1 + 2(1/2) = 2$. Using these values for $t$ and $s$, we find that we indeed have two points of intersection, namely $(1, 1, 1)$ and $(2, 4, 8)$.

There were a couple of interesting solutions: the first involved noticing that in the equations from (1), we can multiply the first by 3 so they each have a $6s$ in common.

$$\begin{aligned}
3t = 3 + 6s &\Leftrightarrow 3t - 3 = 6s \\
&\Rightarrow t^2 = 1 + 3t - 3 && \text{substituting into the second equation} \\
&\Leftrightarrow t^2 - 3t + 2 = 0 \\
&\Leftrightarrow (t-1)(t-2) = 0 \\
&\Leftrightarrow t = 1, 2
\end{aligned}$$

Note that we still need to check that the third equation is satisfied by our solutions for $t$.

The other way of approaching the problem was to notice that the first equation times the second equation gives the third: $t \cdot t^2 = t^3 \Leftrightarrow (1+2s)(1+6s) = 1+14s$. This again leads to solving a quadratic equation, but this time we've used all three equations, so no checking is needed. ☺

**Problem 4.** (0 points) What would you do if someone handed you $1 million?

*Solution.*

1. Pay off student loans.
2. Buy a house in Lafayette so I don't have to pay rent.
3. Put the rest in a low-risk, high-yield savings account so I don't have to stress about money. ☺