MA 261

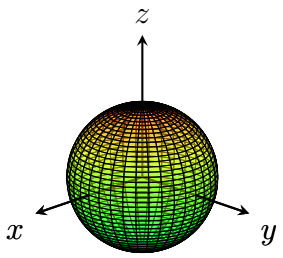
Quiz 4 9 febreiro 2016

Instructions: Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (5 points)[Taken from Stewart 14.1.21] Sketch the domain of the function

$$\sqrt{1-x^2-y^2-z^2}$$

Solution. We require that $1-x^2-y^2-z^2 = 1-(x^2+y^2+z^2) \geq 0$. Equivalently, $x^2+y^2+z^2 \leq 1$, which is the solid unit ball, centered at the origin.



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Problem 2. (5 points)[Taken from Stewart 14.2.10] Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{5y^4\cos^2 x}{x^4+y^4}$$

Solution. We suspect that the limit may not exist. If we approach along the curve $y = x$, the limit becomes

$$\lim_{(x,y)\to(0,0)} \frac{5x^4\cos^2 x}{2x^4} = \lim_{(x,y)\to(0,0)} \frac{5\cos^2 x}{2} = \frac{5}{2},$$

but if we approach along the x-axis, we get

$$\lim_{(x,y)\to(0,0)} \frac{5y^4\cos^2 0}{y^4} = 5.$$

Clearly, $\frac{5}{2} \neq 5$, hence the limit does not exist. ☺

Problem 3. (5 points)[Taken from Stewart 14.3.37] Find all first partial derivatives of the function

$$h(x, y, z, t) = x^2 y \cos(z/t)$$

Proof.

$$\begin{aligned} h_x(x, y, z, t) &= 2xy \cos(z/t) \\ h_y(x, y, z, t) &= x^2 \cos(z/t) \\ h_z(x, y, z, t) &= -\frac{x^2 y}{t} \sin(z/t) \\ h_t(x, y, z, t) &= \frac{x^2 yz}{t^2} \sin(z/t) \end{aligned}$$

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Problem 4. (0 points) What is your favorite thing to do on the weekend?

Solution. I generally try to get caught up on schoolwork just to be behind again on Monday. ☺