MA 261

Quiz 5 16 février 2016

Instructions: Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (5 points) [Similar to 14.4.14] Find the linearization $L(x, y)$ of the function

$$f(x, y) = \sqrt{x + e^4 y}$$

at the point $(3, 0)$.

Solution. The formula for L near (3,0) is given by

$$L(x, y) = f(3, 0) + f_x(3, 0)(x - 3) + f_y(3, 0)(y - 0). \tag{1}$$

Here, $f(3, 0) = \sqrt{3}$, and the partial derivatives are

$$f_x(x, y) = \left. \frac{1}{2\sqrt{x + e^4 y}} \right|_{(x,y)=(3,0)} = \frac{1}{2\sqrt{3}}$$

and

$$f_y(x, y) = \left. \frac{e^4}{2\sqrt{x + e^4 y}} \right|_{(x,y)=(3,0)} = \frac{e^4}{2\sqrt{3}}$$

Now $L(x, y) = \sqrt{3} + \frac{1}{2\sqrt{3}}(x - 3) + \frac{e^4}{2\sqrt{3}} y$. ☺

Problem 2. (5 points) [14.6.12]
Find the directional derivative of

$$f(x, y) = \frac{x}{x^2 + y^2}$$

at the point $(1, 2)$ in the direction of $\mathbf{v} = \langle 3, 5 \rangle$.

Solution. The directional derivative $D_u(1, 2)$ is readily computed by the formula $\nabla f(1, 2) \cdot \mathbf{u}$, where $\mathbf{u}$ is a unit vector in the direction of $\mathbf{v}$. Note that $|\mathbf{v}| = \sqrt{3^2 + 5^2} = \sqrt{34}$, so the $\mathbf{u}$ we want is $\mathbf{u} = \frac{1}{\sqrt{34}} \langle 3, 5 \rangle$. Moreover,

$$\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle = \left\langle \frac{y^2 - x^2}{(x^2 + y^2)^2}, -\frac{2xy}{(x^2 + y^2)^2} \right\rangle,$$

so $\nabla f(1,2) = \left\langle \frac{3}{25}, -\frac{4}{25} \right\rangle$. Now $\nabla f(1,2) \cdot \mathbf{u} = \left\langle \frac{3}{25}, -\frac{4}{25} \right\rangle \cdot \frac{1}{\sqrt{34}} \langle 3, 5 \rangle = \frac{9}{25\sqrt{34}} - \frac{20}{25\sqrt{34}} = -\frac{11}{25\sqrt{34}}$. ☺

Problem 3. (5 points) [14.5.28]
Find dy/dx of the equation

$$\cos(xy) = 1 + \sin y.$$

Solution. Let $F(x,y) = \cos(xy) - \sin y - 1$. Note that $F(x,y) \equiv 0$, so we can use formula (6) on page 929. $F_x(xy) = -y\sin(x,y)$ and $F_y(xy) = -x\sin(xy) - \cos(y)$. Thus,

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = \frac{-y\sin(xy)}{x\sin(xy) + \cos(y)}$$ ☺