MA 261

Quiz 7 7 Μάρτιος 2016

Instructions: Write down your name and section number. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (5 points) Using polar coordinates, set up, but do NOT compute, the integral to find the volume of a sphere of radius a.

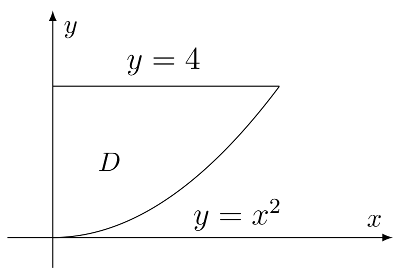
Solution. A sphere of radius a satisfies $x^2+y^2+z^2=a^2$, or $z=\pm\sqrt{a^2-x^2-y^2}$. So by symmetry,

$$\begin{aligned} V &= 2\iint_{x^2+y^2\le a^2} \sqrt{a^2-x^2-y^2}\,dA \\ &= 2\int_0^{2\pi}\int_0^a \sqrt{a^2-r^2}\,r\,dr\,d\theta \end{aligned}$$ ☺

Problem 2. (5 points) Sketch the region of integration and change the order of integration.

$$\int_0^2\int_{x^2}^4 f(x,y)\,dy\,dx.$$

Solution. The region of integration is $D=\{(x,y)\mid x^2\le y\le 4,\ 0\le x\le 2\}$, which is the same as $\{(x,y)\mid 0\le x\le\sqrt{y},\ 0\le y\le 4\}$. Then we have



$$\int_0^2\int_{x^2}^4 f(x,y)\,dy\,dx = \int_0^4\int_0^{\sqrt{y}} f(x,y)\,dx\,dy$$ ☺

Problem 3. (5 points) Evaluate $e^{\pi i}+1$. (Hint: $e^{\pi i}=-1$.)

Solution. $e^{\pi i}+1=-1+1=0.$ ☺

Problem 4. (5 points) Write whatever you want here. Have a good spring break! (And don't forget all the calculus you've learned!)

Solution. This was probably the catchiest response by a student:

"There's 104 days of summer vacation
before school comes along just to end it
so the annual problem for our generation
is finding a good way to spend it..." ☺