# MA 261

Quiz 9 29 March 2016

**Instructions:** Write your name and section number on your quiz. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

**Problem 1.** (6 points) This question was taken from Stewart 16.2.3. Evaluate the line integral

$$\int_C xy^4 \, \mathrm{d}s,$$

where $C$ is the right half of the circle $x^2 + y^2 = 16$.

*Solution.* As stated, the problem does not indicate whether the half-circle is oriented counterclockwise or clockwise. We'll assume the standard counterclockwise. (The reverse orientation will obtain the same answer). We can parametrize $C$ by

$$\mathbf{r}(t) = \langle 4\cos t, 4\sin t \rangle \,,$$

where $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$. Then we have

$$\mathbf{r}'(t) = \langle -4\sin t, 4\cos t \rangle$$

so that

$$|\mathbf{r}'(t)| = \sqrt{(-4\sin t)^2 + (4\cos t)^2} = 4.$$

Now

$$\begin{aligned}
\int_C xy^4 \, \mathrm{d}s &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (4\cos t)(4\sin t)^4 (4) \, \mathrm{d}t \\
&= 4^6 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t \sin^4 t \, \mathrm{d}t \\
&= 4^6 \int_{-1}^{1} u^4 \, \mathrm{d}u \qquad (u = \sin t, \mathrm{d}u = \cos t) \\
&= 4^6 \left[ \frac{1}{5} u^5 \right]_{-1}^{1} = 4^6 \left( \frac{1}{5} + \frac{1}{5} \right) = \frac{2 \cdot 4^6}{5}
\end{aligned}$$

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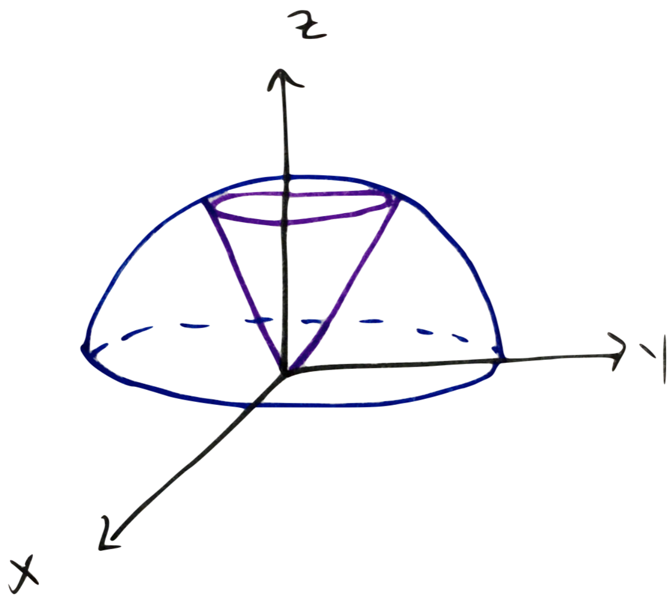
**Problem 2.** (6 points) Calculate the volume of the solid inside $x^2 + y^2 + z^2 = 4$, below $z = \sqrt{x^2 + y^2}$ and above the $xy$-plane.

*Solution.* The Region we're interested in is inside the upper hemisphere of radius 2 and below the cone that sits inside it.




Recall that the cone $z = \sqrt{x^2+y^2}$ in spherical coordinates is given by $\varphi = \frac{\pi}{4}$. So

$$E = \left\{ (\rho, \theta, \varphi) \mid 0 \le \rho \le 2,\ 0 \le \pi \le 2\pi,\ \frac{\pi}{4} \le \varphi \le \frac{\pi}{2} \right\}.$$

Now the integral that represents this volume is

$$\begin{aligned}
\int_{\pi/4}^{\pi/2} \int_0^{2\pi} \int_0^2 \rho^2 \sin\varphi \, \mathrm{d}\rho \, \mathrm{d}\theta \, \mathrm{d}\varphi &= \int_{\pi/4}^{\pi/2} \sin\varphi \, \mathrm{d}\varphi \int_0^{2\pi} \mathrm{d}\theta \int_0^2 \rho^2 \, \mathrm{d}\rho \\
&= -\cos\varphi\Big|_{\varphi=\pi/4}^{\varphi=\pi/2} \cdot 2\pi \cdot \frac{1}{3}\rho^3 \Big|_{\rho=0}^{\rho=2} = \frac{1}{\sqrt{2}} \cdot 2\pi \cdot \frac{8}{3} \\
&= \frac{16\pi}{3\sqrt{2}}.
\end{aligned}$$

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**Problem 3.** (3 points) What is your section number?
(11:30 = 241, 12:30 = 324, 1:30 = 338)