MA 261 Midterm Exam 2 Summer 2018

Name: Solutions

ID number: ______________________

Grade	Cut off	Number
A	≥ 80	7
B	≥ 65	12
C	≥ 50	11
D	≥ 40	2
F	< 40	2

Instructions:

1. This is a one-hour exam.
2. There are 10 problems on this exam.
3. No books, notes, or calculators are allowed.
4. Please turn off your cell phone.
5. Circle one and only one choice for each multiple-choice problem. No partial credit will be given for multiple-choice problems.
6. Show all relevant work on non-multiple-choice problems. Partial credit will be given for steps leading to the correct solutions. Write your final answer in the box provided.
7. You may use a writing utensil, your own brain and the paper provided in this exam. Use of any other persons or resources will be considered cheating and will be reported to the Office of the Dean of Students.

Low: 34.5
High: 97
mean: 65.54
median: 67

I agree to abide by the instructions above:

Signature: ______________________

Page	Score	Points Possible
2		18
3		16
4		11
5		11
6		11
7		11
8		11
9		11
Total		100

1. (10 points) Consider the table below. Classify each critical point, or if a point is not a critical point, circle "Not a CP."

(a,b)	$g(a,b)$	$g_x(a,b)$	$g_y(a,b)$	$g_{xx}(a,b)$	$g_{xy}(a,b)$	$g_{yy}(a,b)$
$(0,1)$	0	3	0	0	-2	4
$(4,3)$	-3	0	0	-1	2	-6
$(2,7)$	15	0	0	4	5	8
$(5,6)$	4	0	0	3	5	2

$(0,1)$: $g_x = 3 \neq 0$

Not a CP local maximum local minimum saddle point

$(4,3)$: $D = (-1)(-6) - 2^2 = 6-4 > 0$, $g_{xx} < 0$

Not a CP local maximum local minimum saddle point

$(2,7)$: $D = (4)(8) - 5^2 = 32-25 > 0$, $g_{xx} > 0$

Not a CP local maximum local minimum saddle point

$(5,6)$: $D = (3)(2) - 5^2 < 0$

Not a CP local maximum local minimum saddle point

2. (8 points) Find the directional derivative of the function $f(x,y) = x^4 - x^2y^3$ at the point $(2,1)$ in the direction of $\mathbf{v} = \langle 1, 1 \rangle$.

A. 8

B. $8\sqrt{2}$

C. $12\sqrt{2}$

D. $16\sqrt{2}$

E. 16

$\vec{u} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\nabla f = \langle 4x^3 - 2xy^3, -3x^2y^2 \rangle$

$\nabla f(1,2) = \langle 28, -12 \rangle$

$D_{\vec{u}} = \nabla f(1,2) \cdot \vec{u} = \frac{28}{\sqrt{2}} - \frac{12}{\sqrt{2}} = \frac{16}{\sqrt{2}} = 8\sqrt{2}$

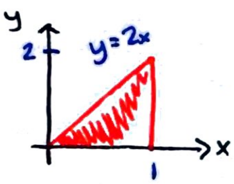
3. (8 points) Compute the integral

$$\int_0^2 \int_{y/2}^1 e^{x^2}\, dx dy = \int_0^1 \int_0^{2x} e^{x^2}\, dy\, dx$$

$$= \int_0^1 2x e^{x^2}\, dx$$

$$= e^{x^2}\Big|_0^1$$

$$= e - 1$$



A. 1

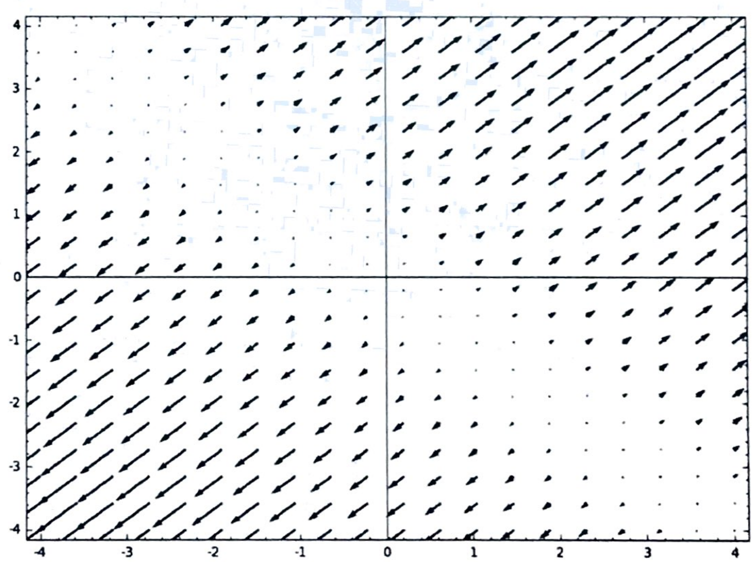
B. e

C. $e - 1$

D. $1 - e$

E. This integral cannot be computed.

4. (8 points) Choose the function f whose gradient is plotted below.



A. $f(x, y) = \sin(x^2 + y^2)$ $\Rightarrow \nabla f = \langle 2x\cos(x^2+y^2), 2y\cos(x^2+y^2)\rangle$

B. $f(x, y) = x^2 + y^2$ $\Rightarrow \nabla f = \langle 2x, 2y\rangle$

C. $f(x, y) = 2x + 2y$ $\Rightarrow \nabla f = \langle 2, 2\rangle$

D. $f(x, y) = (x + y)^2$ $\rightarrow \nabla f = \langle 2(x+y), 2(x+y)\rangle$

E. $f(x, y) = x^2 + xy$ $\Rightarrow \nabla f = \langle 2x+y, x\rangle$

Should vanish along $y = -x$, have slope 1 elsewhere

5. (11 points) Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

$$f(x,y) = xy \Rightarrow f_x = y,\ f_y = x$$

$$A = \iint_{x^2+y^2 \le 1} \sqrt{1+x^2+y^2}\, dA$$

$$= \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^1 \sqrt{1+r^2}\, r\, dr$$

$$= \int_0^{2\pi} d\theta \int_1^2 \tfrac{1}{2} u^{\frac{1}{2}}\, du$$

$$= 2\pi \cdot \tfrac{1}{2} \cdot \tfrac{2}{3} \cdot u^{3/2} \Big|_1^2$$

$$= \tfrac{2\pi}{3}\left(2^{3/2} - 1^{3/2}\right)$$

$$\boxed{\tfrac{2\pi}{3}\left(2\sqrt{2} - 1\right)}$$

6. (11 points) Use the method of Lagrange multipliers to find the **minimum** value of the function $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $x + y + z = 12$.

$g(x,y,z) = x+y+z$

$\nabla f = \lambda \nabla g$

$$\left.\begin{aligned} 2x &= \lambda \\ 2y &= \lambda \\ 2z &= \lambda \end{aligned}\right\} \Rightarrow x = y = z$$

$12 = x+y+z = x+x+x \Rightarrow x = y = z = 4.$

$f(4,4,4) = 3 \cdot 4^2 = 48$

$\boxed{48}$

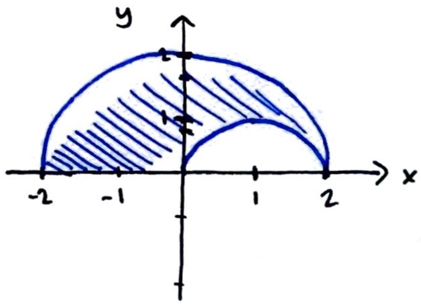
7. (10 points) Set up **but do not compute** an integral in polar coordinates that represents the area of the region between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 2x$ that lies above the line $y = 0$.

$$x^2+y^2=2x \iff x^2-2x+1+y^2=1$$

$$\iff (x-1)^2+y^2=1$$

$$\iff r^2 = 2r\cos\theta$$

$$\iff r = 2\cos\theta$$



$$2\cos\theta \le r \le 2$$

$$0 \le \theta \le \pi$$

$$\boxed{\int_0^{\pi}\int_{2\cos\theta}^{2} r\,dr\,d\theta}$$

8. (10 points) Set up **but do not compute** an integral in cylindrical coordinates that represents $\iiint_E (x^2 z + y^2 z)\, dV$, where E is the region bounded by the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$.

$z = x^2 + y^2 \iff z = r^2$

$x^2 + y^2 + z^2 = 2 \iff r^2 + z^2 = 2$

$r^2 \le z \le \sqrt{2 - r^2}$

$0 \le r \le 1$

$0 \le \theta \le 2\pi$

Sphere and paraboloid intersect when

$2 = z^2 + x^2 + y^2$

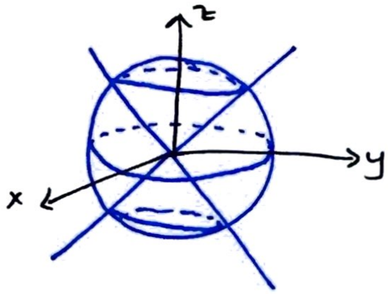
$2 = z^2 + z \implies z = 1$

So $x^2 + y^2 = 1 \implies r = 1$

Also, $x^2 z + y^2 z = (x^2 + y^2) z = r^2 z$

$$\boxed{\int_0^{2\pi} \int_0^1 \int_{r^2}^{\sqrt{2-r^2}} r^3 z \, dz\, dr\, d\theta}$$

9. (10 points) Set up but **do not compute** an integral in spherical coordinates that represents the volume of the region inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cone $z^2 = x^2 + y^2$.



$\frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}$

$0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq 3$

$dV = \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$

$$\int_0^{2\pi} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_0^3 \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

10. (10 points) Evaluate the integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the part of the parabola $x = 9 - y^2$ from $(5, -2)$ to $(0, 3)$ and $\mathbf{F}(x, y) = \langle y^2, x \rangle$.

$\vec{r}(t) = \langle 9 - t^2, t \rangle, \quad -2 \le t \le 3$

$\vec{r}\,'(t) = \langle -2t, 1 \rangle, \quad \vec{F}(\vec{r}(t)) = \langle t^2, 9 - t^2 \rangle$

$$\int_C \vec{F} \cdot d\vec{r} = \int_{-2}^{3} \langle t^2, 9 - t^2 \rangle \cdot \langle -2t, 1 \rangle \, dt$$

$$= \int_{-2}^{3} \left(-2t^3 + 9 - t^2 \right) dt$$

$$= \frac{-1}{2} t^4 + 9t - \frac{1}{3} t^3 \Big|_{-2}^{3}$$

$$= \frac{-1}{2}(3)^4 + 9(3) - \frac{1}{3}(3)^3 - \left[\frac{-1}{2}(-2)^4 + 9(-2) - \frac{1}{3}(2)^3 \right]$$

$$= \frac{-45}{2} - \left(-\frac{70}{3} \right)$$

$$= 5/6$$

$$\boxed{\frac{5}{6}}$$