Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. Find an equation of the tangent plane to

$$\mathbf{r}(u,v) = \left\langle u-2v, u^2-4v, v^2-2v \right\rangle$$

at the point $(0,0,-1)$.

Solution. We compute $\mathbf{r}_u = \langle 1, 2u, 0\rangle$ and $\mathbf{r}_v = \langle -2, -4, 2v-2\rangle$, so

$$\mathbf{r}_u \times \mathbf{r}_v = \langle 2u(2v-2), 2-2v, -4+4u\rangle .$$

At the point $(0,0,-1)$, we must have $u = 2v$, which means that $u^2 - 4v = u^2 - 2u = 0$, so $u = 0$ or $u = 2$. Since $v \neq 0$ (looking at the third component of $\mathbf{r}(u,v)$), we must have $u = 2$, $v = 1$. Plugging this into $\mathbf{r}_u \times \mathbf{r}_v$, we find a normal vector to the tangent plane to be $\mathbf{n} = \langle 0,0,4\rangle$. Thus an equation of the tangent plane at the point $(0,0,-1)$ is

$$0 + 0 + 4(z+1) = 0,$$

or $z = -1$. □

Problem 2. Evaluate the surface integral

$$\iint_S (x^2+y^2+z)\, dS,$$

where S is the part of the graph of the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane.

Solution. We parametrize the surface S as a graph with $g(x,y) = 4 - x^2 - y^2$ for $(x,y) \in D$, where $D = \{(x,y) \mid x^2+y^2 \leq 4\}$. Then

$$\begin{aligned}
\iint_S (x^2+y^2+z)\, dS &= \iint_D (x^2+y^2+z)\, |\mathbf{r}_x \times \mathbf{r}_y|\; dA \\
&= \iint_D (x^2+y^2+z)\sqrt{\left(-\frac{\partial g}{\partial x}\right)^2 + \left(-\frac{\partial g}{\partial y}\right) + 1}\, dA \\
&= \iint_D (x^2+y^2+\underbrace{4-x^2-y^2}_{z})\sqrt{4x^2+4y^2+1}\, dA \\
&= \int_0^{2\pi}\int_0^2 4\sqrt{4r^2+1}r\, dr d\theta \\
&= \int_0^{2\pi} d\theta \int_1^{17} \frac{1}{2} u^{1/2}\, du \\
&= \frac{2\pi}{3}(17\sqrt{17}-1)
\end{aligned}$$

□