Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Its acceleration is $\mathbf{a}(t) = 6t\mathbf{i} + 12t^2\mathbf{j} - 6t\mathbf{k}$. Find the position function.

Solution. Since $\mathbf{r}''(t) = \mathbf{v}'(t) = \mathbf{a}(t)$, we just need to take two different anti-derivatives and use the initial conditions $\mathbf{r}(0) = \langle 0,0,0\rangle$ and $\mathbf{v}(0) = \langle 1,-1,3\rangle$. Now

$$\mathbf{v}(t) = \left\langle 3t^2, 4t^3, -3t^2\right\rangle + \mathbf{C},$$

and

$$\langle 1,-1,3\rangle = \mathbf{v}(0) = \langle 0,0,0\rangle + \mathbf{C}$$

implies that $\mathbf{C} = \langle 1,-1,3\rangle$ and $\mathbf{v}(t) = \left\langle 3t^2+1, 4t^3-1, -3t^2+3\right\rangle$. Next

$$\mathbf{r}(t) = \left\langle t^3+t, t^4-t, -t^3+3t\right\rangle + \mathbf{D}.$$

Finally since $\mathbf{r}(0) = \mathbf{0}$, we find $\mathbf{D} = \mathbf{0}$, so we conclude

$$\mathbf{r}(t) = \left\langle t^3+t, t^4-t, -t^3+3t\right\rangle.$$

□

Problem 2. Let $f(x,y) = \sqrt{4-x^2-y^2} + \sqrt{1-x^2}$.

(a) Evaluate $f(1,\sqrt{3})$.

(b) Find and Sketch the domain of f.

(c) Find the range of f.

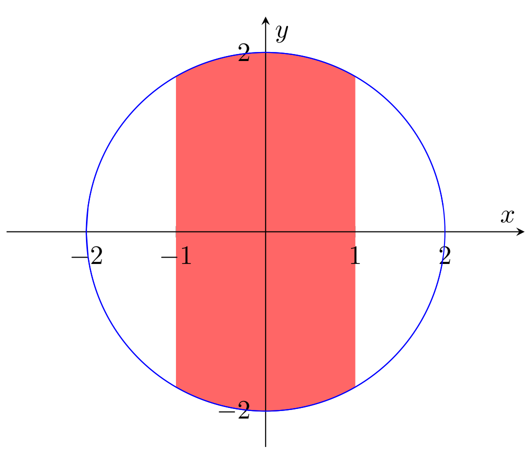
Solution.

(a) $f(1,\sqrt{3}) = \sqrt{4-1-3} + \sqrt{1-1} = 0$.

(b) The first square root requires that $4-x^2-y^2 \geq 0$, or $x^2+y^2 \leq 4$. The second requires that $x^2 \leq 1$. This is the area in the disk of radius 2 centered at the origin with x-values between -1 and 1. In set notation, this is

$$D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 4, x^2 \leq 1\}.$$

The picture is given below.



(c) For the range, note that $x^2 + y^2 \geq 0$ and $x^2 \geq 0$. This means that $4 - (x^2 + y^2) \leq 4$ and $1 - x^2 \leq 1$, so

$$\sqrt{4 - x^2 - y^2} + \sqrt{1 - x^2} \leq \sqrt{4} + \sqrt{1} = 3.$$

And of course each square root is bounded below by 0. So the range is $[0, 3]$. $\square$