Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. (10 points) Find the maximal rate of change in $f(x, y) = 5 \ln(x + 2y)$ at (1,2) and the direction in which it occurs.

Solution. The the maximal rate of change at $(1, 2)$ is $|\nabla f(1, 2)|$. Since

$$\nabla f(x, y) = \left\langle \frac{5}{x + 2y}, \frac{10}{x + 2y} \right\rangle,$$

we have $\nabla f(x, y) = \langle 1, 2 \rangle$. So $|\nabla f(1, 2)| = \sqrt{5}$ and the direction in which it occurs is $\mathbf{u} = \frac{1}{\sqrt{5}} \langle 1, 2 \rangle$. □

Problem 2. (10 points) Calculate the iterated integral $\int_0^1 \int_0^1 y e^{xy} dx dy$.

Solution.

$$\begin{aligned} \int_0^1 \int_0^1 y e^{xy} dx dy &= \int_0^1 \Big[e^{xy} \Big]_{x=0}^{x=1} dy \\ &= \int_0^1 (e^y - 1)\, dy \\ &= e^y - y \Big|_0^1 \\ &= e - 2 \end{aligned}$$

□