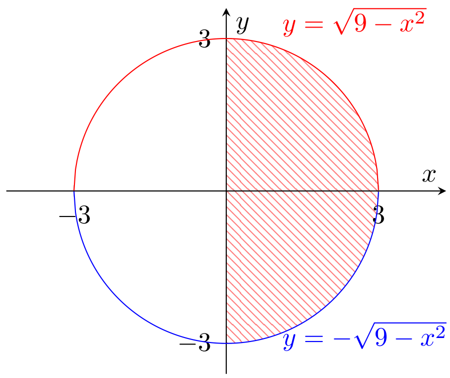
Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

Problem 1. Evaluate the iterated integral by converting to polar coordinates.

$$\int_0^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 - xy^2) dy dx$$

Solution. The region of integration is



In polar coordinates we have $x = r\cos\theta$ and $y = r\sin\theta$. So the region D is $D = \{(r,\theta) \mid 0 \le r \le 3, -\pi/2 \le \theta \le \pi/2\}$, and the integral becomes

$$\begin{aligned}
\iint_D (x^3 - xy^2)\, dA &= \int_{-\pi/2}^{\pi/2} \int_0^3 \left(r^3\cos^3\theta - r^3\cos\theta\sin^2\theta \right) r\, dr d\theta \\
&= \int_{-\pi/2}^{\pi/2} \left(\cos^3\theta - \cos\theta\sin^2\theta \right) d\theta \int_0^3 r^4\, dr \\
&= \int_{-\pi/2}^{\pi/2} \left((1-\sin^2\theta)\cos\theta - \cos\theta\sin^2\theta \right) d\theta \left[\frac{1}{5} r^5 \right]_0^3 \\
&= \frac{3^5}{5} \left(\int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta - 2\int_0^{2\pi} \cos\theta\sin^2\theta\, d\theta \right) \\
&= \frac{3^5}{5} \left(\left[\sin\theta \right]_{-\pi/2}^{\pi/2} - \frac{2}{3} \left[\sin^3\theta \right]_{-\pi/2}^{\pi/2} \right) \\
&= \frac{3^5}{5} \left(2 - \frac{2}{3} \cdot 2 \right) \\
&= \frac{2 \cdot 3^4}{5} = \frac{162}{5}
\end{aligned}$$

□

Problem 2. Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$.

Solution. We have $f(x,y) = xy$, so $f_x = y$ and $f_y = x$. So the area for $x^2+y^2 \leq 1$ is given by

$$\begin{aligned}
\iint_{x^2+y^2\leq 1} \sqrt{f_x^2 + f_y^2 + 1}\, dA &= \iint_{x^2+y^2\leq 1} \sqrt{x^2+y^2+1}\, dA \\
&= \int_0^{2\pi} \int_0^1 \sqrt{r^2+1} \cdot r\, dr d\theta \\
&= \int_0^{2\pi} d\theta \int_1^2 \frac{1}{2} u^{1/2}\, du \\
&= \frac{2\pi}{3}(2\sqrt{2} - 1)
\end{aligned}$$

□