Instructions. Show all work, with clear logical steps. No work or hard-to-follow work will lose points.

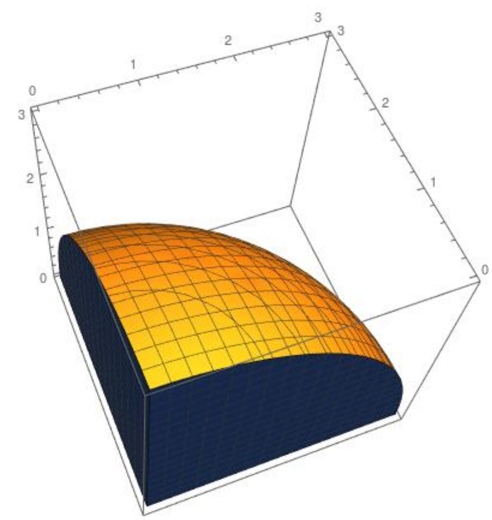
Problem 1. (10 points) Convert the following to an equivalent triple integral in spherical coordinates and evaluate.

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} xz \, dzdydx$$

Solution. The integrand in spherical coordinates is

$$\rho \sin\phi \cos\theta \cdot \rho\cos\phi \cdot \rho^2 \sin\phi \, d\rho d\phi d\theta = \rho^4 \sin^2\phi \cos\phi \cos\theta \, d\rho d\phi d\theta$$

The region is the part of the sphere of radius 3 in the positive octant:



So $0 \le \theta \le \pi/2$, $0 \le \rho \le 3$ and $0 \le \phi \le \pi/2$. So the integral becomes

$$\begin{aligned}
\int_0^{\pi/2}\int_0^{\pi/2}\int_0^3 \rho^4 \sin^2\phi\cos\phi\cos\theta \, d\rho d\phi d\theta &= \int_0^3 \rho^4 \, d\rho \int_0^{\pi/2} \sin^2\phi \cos\phi \, d\phi \int_0^{\pi/2} \cos\theta \, d\theta \\
&= \int_0^3 \rho^4 \, d\rho \int_0^1 u^2 \, d\phi \int_0^{\pi/2} \cos\theta \, d\theta \\
&= \left[\frac{1}{5}\rho^5\right]_0^3 \left[\frac{1}{3}u^3\right]_0^1 \left[\sin\theta\right]_0^{\pi/2} \\
&= \frac{243}{5} \cdot \frac{1}{3} \\
&= \frac{81\pi}{5}
\end{aligned}$$

□

Problem 2. (10 points) Find the gradient vector field of f.

(a) $f(x,y,z) = x^2 y e^{\sin(yz)}$.

(b) $f(x,y) = \cos(\sqrt{x^2+y^2})$.

Solution. We just use the chain rule several times.

(a) $\nabla f = \left\langle 2xye^{\sin(yz)}, x^2e^{\sin(yz)} + x^2yz\cos(yz)e^{\sin(yz)}, x^2y^2\cos(yz)e^{\sin(yz)} \right\rangle$

(b) $\nabla f = \left\langle -\frac{x\sin\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}, -\frac{y\sin\left(\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}} \right\rangle$ □