



FIGURE 9

but the traces in the  $xz$ - and  $yz$ -planes are the hyperbolas

$$\frac{x^2}{4} - \frac{z^2}{4} = 1 \quad y = 0 \quad \text{and} \quad y^2 - \frac{z^2}{4} = 1 \quad x = 0$$

This surface is called a **hyperboloid of one sheet** and is sketched in Figure 9.

The idea of using traces to draw a surface is employed in three-dimensional graphing software for computers. In most such software, traces in the vertical planes  $x = k$  and  $y = k$  are drawn for equally spaced values of  $k$ , and parts of the graph are eliminated using hidden line removal. Table 1 shows computer-drawn graphs of the six basic types of quadric surfaces in standard form. All surfaces are symmetric with respect to the  $z$ -axis. If a quadric surface is symmetric about a different axis, its equation changes accordingly.

TABLE 1 Graphs of quadric surfaces

Surface	Equation	Surface	Equation
<p>Ellipsoid</p>	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ <p>All traces are ellipses. If <math>a = b = c</math>, the ellipsoid is a sphere.</p>	<p>Cone</p>	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ <p>Horizontal traces are ellipses. Vertical traces in the planes <math>x = k</math> and <math>y = k</math> are hyperbolas if <math>k \neq 0</math> but are pairs of lines if <math>k = 0</math>.</p>
<p>Elliptic Paraboloid</p>	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ <p>Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.</p>	<p>Hyperboloid of One Sheet</p>	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ <p>Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.</p>
<p>Hyperbolic Paraboloid</p>	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ <p>Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where <math>c &lt; 0</math> is illustrated.</p>	<p>Hyperboloid of Two Sheets</p>	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ <p>Horizontal traces in <math>z = k</math> are ellipses if <math>k &gt; c</math> or <math>k &lt; -c</math>. Vertical traces are hyperbolas. The two minus signs indicate two sheets.</p>