MATH 511, Spring 2018, Final exam

NAME:

1. How many positive eigenvalues does this matrix have?

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}.$$

2. All these matrices have eigenvalues 3 and 6. Which of them are diagonalizable? Give a *short justification* of your answer.

$$A = \begin{pmatrix} 5 & -2 & 0 \\ -1 & 4 & 0 \\ -1 & 1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 5 & 1 \\ 3 & -1 & 4 \end{pmatrix},$$

$$C = \begin{pmatrix} 5 & -3 & 1 \\ -1 & 3 & 1 \\ -1 & 0 & 4 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}.$$

3. For which real t is the matrix

$$\begin{pmatrix} 1 & t & 0 \\ t & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

positive definite?

4. Find $\exp(At)$ for

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

5. Solve the equation

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 2-x & 1 \\ 1 & 1 & 1 & 3-x \end{vmatrix} = 0.$$

6. The system

$$x' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{x}$$

A. does not have any non-zero solution

B. has only periodic solutions

C. has only unbounded solutions

D. has only solutions which tend to 0 as $t \to +\infty$

E. has some unbounded solutions

7. The matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

has 0 as an eigenvalue. The other eigenvalues of this matrix are

A. Distinct positive

B. Distinct negative

C. One positive and one negative

D. Multiple positive

E. Multiple negative

8. Let A and B be real $n \times n$ matrices. Which of the following statements are alwats true?

A. If $A^T A = AA^T$ then A is invertible

B. If $A^T A = AA^T$ then A is orthogonal

C. If $A^T A = AA^T$ then A is diagonalizable

D. If $A = B^T B$ then A is positive definite

E. $e^{A+B} = e^A e^B$.

9. The signature of the quadratic form with matrix

$$\begin{pmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

is

A. $(+,+,-)$

B. $(+,-,-)$

C. $(-,-,-)$

D. $(+,-,0)$,

E. none of the above.

10. Let U, V, W be subspaces of $\mathbf{R}^{10}$, and

$$\dim U = \dim V = \dim W = 7.$$

What is the smallest possible dimension of $U \cap V \cap W$?

A. 0
B. 1
C. 2
D. 3
E. 4