MA/STAT 416 - PROBABILITY

Sections 005 & 006 (CRNs 15620 & 15625)

005: MWF 3:30-4:20

006: MWF 2:30-3:20

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PURDUE UNIVERSITY | Department of Mathematics

Lecture 1.1

Part 1: Logistics & Overview

Note: today is an exception, but we will only do lectures from slides for ~20 minutes most days. (And some days none at all!) Most of most days will involve solving practice problems and explaining theory through examples.

Course resources

They’re in a few different places

- Course homepage/syllabus: https://www.math.purdue.edu/~esampert/416/
- Brightspace: only used for announcements, BoilerCast & gradebook (more-or-less)
- Detailed course calendar (bookmark this!):
  https://www.math.purdue.edu/~esampert/416/cal
- Textbook: Ross’s *Probability*, 10^{th} ed. Pearson. (Don’t need MyLab.)
- My office hours:
  - Mondays: 4:30-5:30 in my office, MATH 706
  - Thursdays: 7-8pm over Zoom. Zoom link will be sent in an announcement shortly after class.
  - Fridays: 1:15-2:15 in my office, MATH 706
- Stats Help Room: I will provide more info once I get it.
- Sitting right next to you (your classmates ☺).

How you will be assessed

Four types of grades in this class

- Homework: due weekly at beginning of class on Friday. ~13 assignments total. ~6 problems per assignment. 2 lowest dropped. No late work accepted (for the most part). First assignment due next week on Friday, 1/24
- Two midterm exams. Both in-class. MT1 is 2/17, MT2 is 4/7.
- One final exam. In-class, comprehensive, up to 2 hours long. Schedule TBD by Purdue Registrar.
- In-class draft. Details to be explained momentarily. (It’s not as complicated as it looks on the syllabus.)

In-class draft	3%
HW	30%
MT1	16%
MT2	16%
Final	35%

How to succeed in this class

In order of priority

1. Do your homework in a timely manner (including daily draft problem).

2. Do your required reading (ideally before coming to class).

3. Communicate with me! Talk to me in/before/after class/office hours or over email. **In particular, come to class!**

4. Come to my office hours or go to Stats Help Room, especially if you need help getting started on a problem or need help understanding why you didn't get full credit for something.

5. Keep up with all course communications (including the feedback you get on homework and studying info/tips I will provide for the exams; pay attention to announcements and regularly refresh the course calendar).

QUESTIONS ABOUT LOGISTICS OR HOW TO SUCCEED?

LET'S DISCUSS THE "DRAFT"

https://televisiontunes.com/The_Price_is_Right___Main.html#google_vignette

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In-class draft rules are on the syllabus/homepage:
https://www.math.purdue.edu/~esampert/416/

The in-class draft is supposed to be a ***low stakes*** and ***fun*** way to keep everyone ***accountable*** for maintaining a ***steady study schedule*** for this class.

It is admittedly a bit experimental. If at any point it stops being all of these things, then *please let me know.*

NOW: WHAT IS THIS CLASS ALL ABOUT?

Probability!... But... what is probability?

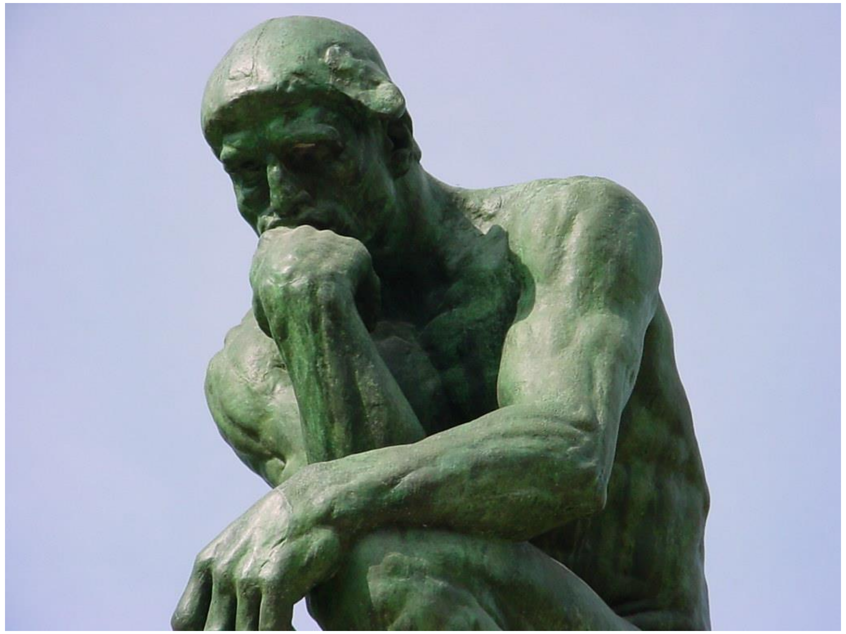
Several possible answers we might give, depending if we want to be "narrow" or "broad."

- Extremely narrow mathematical answer: a probability is any number p such that $0 \leq p \leq 1$.

- Extremely broad "philosophical" answers (which are matters of *interpretation*, NOT "mathematical facts"):
  - Frequentist perspective: a probability is a mathematical representation of the ratio of different outcomes we see in an idealized experiment that we can repeat an infinite number of times
  - realism: a probability is some kind of intrinsic real property of a system (often called "propensity" in philosophy) that causes it to choose different outcomes according to some internal rules (kind of a lame answer if you ask me)
  - Bayesianism: a probability is a quantification of belief about whether or not something is true (1) or false (0). (Who's belief?!)

My preferred philosophy (Bayesianism)

Probability is an (apparently consistent) mathematical theory about how to *think rationally* in the presence of *uncertainty*.

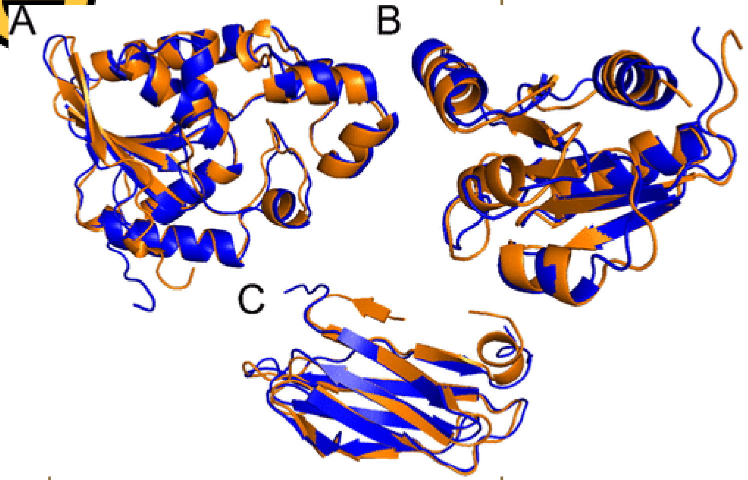


As such, probability is the branch of math that is applied most often outside of mathematics (well, besides basic arithmetic…). It is of fundamental importance anywhere *inductive inferences* need to be made. For example:

- **Experimental sciences.** "How sure can I be that the outcomes of my experiments confirm my hypothesis?"
- **Statistical modeling and forecasting.** "What is the probability that 'the big one' (earthquake) happens in Los Angeles in the next 10 years?" Nate Silver's election forecast. "Should I short Tesla?" "Will the sun rise tomorrow?"
- **Machine learning/AI.** "How can I best train an AI to use a given data set in order to make new discoveries?"

It's also important in a somewhat different way that is rather specific to computer science:

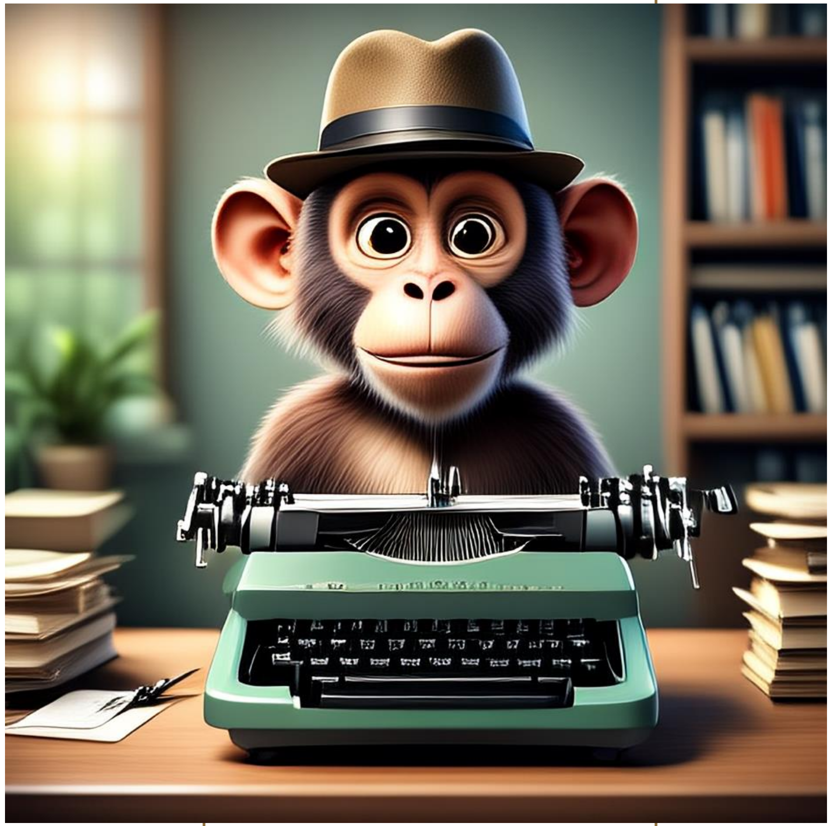
- **Design of algorithms & cryptosystems.** "How can I use randomness in order to maintain and share secrets securely?" "How are quantum computers better than non-quantum computers?"



Example: gambler's ruin

Suppose you have $1,000. Is it reasonable to agree to play the following game with me?

- I flip a fair coin. If it lands heads, then you give me $100. If it lands tails, then I give you $100.
- Repeat until one of us runs out of money.
- I have as much money as a small operation casino owner, let's say ~$1 million USD.



Why study probability?

I have already given you several implicit reasons, but let’s make them explicit:

- Probability theory is a mathematically consistent and interesting set of rules, so we might “play the game” just for its own sake.

- The laws of probability appear to conform to our reality in some sense, and are thus worthy of study. In scenarios where it is possible to build models that correctly account for all variables, making predictions in a way that is not in line with the principles of probability leads to… incorrect predictions!

- Put another way: the mathematics of probability provides a framework for thinking rationally in the face of uncertainty.

- Given the previous: we can use the mathematics of probability as a tool to make money, win games, or learn new things about the world.

WHAT DO YOU THINK?

Does anybody need more convincing about whether they should take this class? Are there any reasons/applications you like that I missed?

Let me frank about something: in this class, we will not “really” do most of the things I mentioned on previous slides (profit/win/discover new science). However, the things you learn this semester could be foundational to your eventually accomplishing them in your own careers.

Getting down to brass tacks I

By the time the semester ends, if you are successful in this class then you will be able to do all of the following:

- Explain the formal properties that all probabilistic systems share, using mathematically precise language.
- Use Bayes's theorem to explain how probabilities should be updated in light of new evidence.
- Formulate mathematically precise constructions out of various intuitive ideas, including "expected value," "variance," "covariance," "correlation," "independence," and the "law of large numbers."
- Understand several important examples of probabilistic systems, both of discrete and continuous types.
- Have some familiarity with the reasons why certain probabilistic systems (like bell curves) are so prevalent throughout science.

Getting down to brass tacks II

Our course has 3 parts.

- **Part 1: Fundamentals of probability.** Chapters 1-3 of Ross. MT1 after this.
  - Topics: combinatorics ("fancy counting tricks"), laws of probability, conditional probabilities, Bayes's theorem.

- **Part 2: ~~(Real valued) random variables.~~ Tons of examples.** Chapters 4-5 of Ross. MT2 after this.
  - Topics: random variables, expectation value, variance, discrete random variables, continuous random variables

- **Part 3: advanced topics.** Chatpers 6-8.
  - Topics: jointly distributed random variables, conditional distributions, correlation, covariance, central limit theorem

Possible difficulties on the horizon

- **Part 1**: learning to be precise and rigorous about seemingly ”obvious” things. Lots of new notation.

- **Part 2:** only a few general principles we need to grapple with, but LOTS of examples. I expect it will be non-trivial to keep them all straight. (Exam “cheat sheet” will help with this.) Continuous random variables require comfort with calculus as a prerequisite, whereas discrete random variables require combinatorics.

- **Part 3:** new, more subtle concepts being stacked on top of combinatorics or calculus.

ANY QUESTIONS ABOUT WHERE WE’RE HEADED THIS SEMESTER?

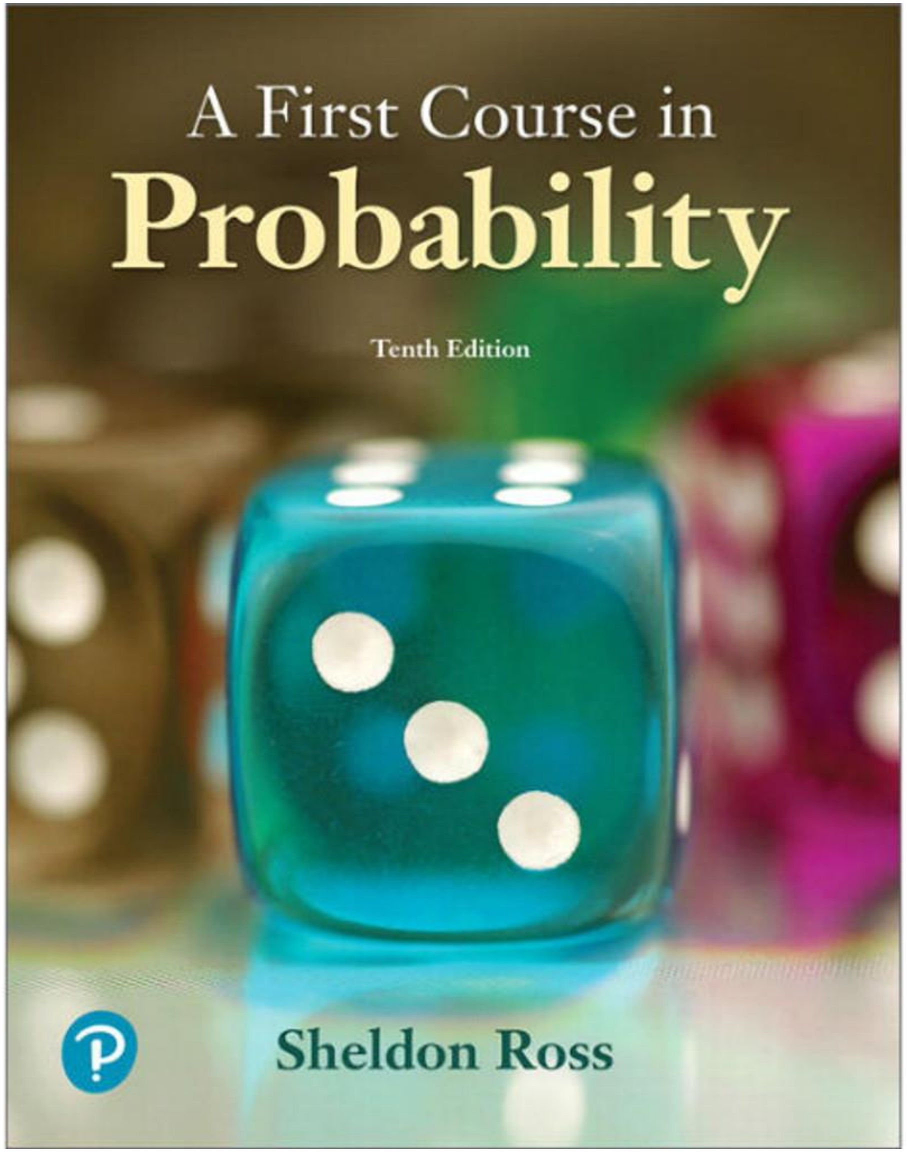
BOILER UP

Everything we just discussed on the slides up to here was for your benefit, just to get you oriented. The real stuff starts now.

Lecture 1.1

Part 2: starting with combinatorics

The book says "combinatorial analysis." Most people just say "combinatorics." They are synonyms.



Today's reading:

- **Table of contents**
- **Preface**
- **Sections 1.1-1.2 of Ross.**

Next time:

- **1.3-1.4**

Combinatorics in a nutshell

Combinatorics is sometimes (jokingly) called "advanced counting."

- Branch of mathematics (separate from probability theory) that is all about counting or bounding the sizes of various complicated sets.

- Example (that we'll revisit later): how many 5 card hands (out of a standard deck of 52 cards) show a full house? (pair+3 of a kind)

- Important in probability theory both historically (Gerolamo Cardano's 16th century description of "games of chance") and as a source of examples ("discrete" random variables)

The basic principle of counting I

As worded in the book:

Suppose that two experiments are to be performed. Then if experiment 1 can result in any one of m possible outcomes and if, for each outcome of experiment 1, there are n possible outcomes of experiment 2, then together there are mn possible outcomes of the two experiments.

The basic principle of counting II

The word “experiment” isn’t very important. E.g.:

Suppose Alex and Blake are planning a picnic. They agree that Alex will bring the main course and Blake will bring the dessert. They also agree that Alex will choose 1 option for the main course out of a list of 13 they have narrowed it down to, and Blake will choose 1 option for the dessert out of a list of 7 they have narrowed it down to. How many possible menus could their picnic consist of?

The basic principle of counting III

We can iteratively apply the same principle if we have more than 2 experiments.

Suppose that Casey decides to join Alice and Blake at the picnic. They agree that Casey will bring a salad, choosing from a list of 5 possible choices. How many possible menus could the 3-course picnic consist of?

Draft problem

To be presented by the draftee determined at the beginning of class on Wednesday.

A standard Indiana license plate always consists of 3 numbers (0-9) followed by 3 letters from the standard Latin/English alphabet (A-Z, capitalized).

(a) How many possible license plates are there?
(b) What if the letters can be in either lower or upper case?
(c) What if no number or letter can be repeated (and we only use upper case)?