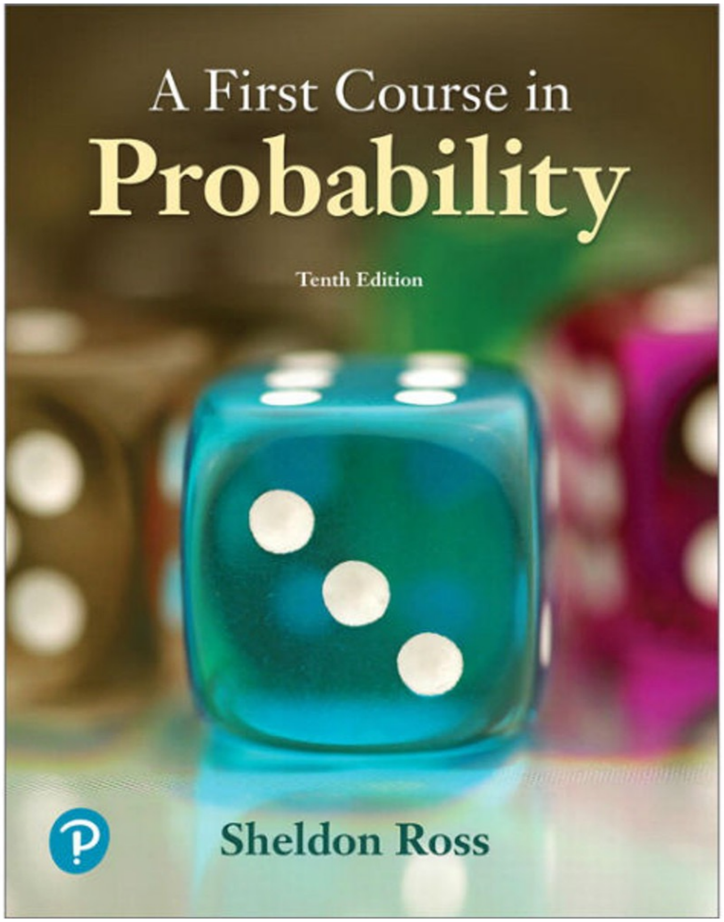
Lecture 10.3

PDFs of functions of random variables

PURDUE UNIVERSITY | Department of Mathematics



Today's reading: 5.7

Next class: 6.1 (Not on MT2)

HW8 due now.

HW9 now available. Due Friday.

HW10 will be assigned in two weeks (due to MT2).

I'll do my best to get practice MT2 and studying recommendations to you before class on Wednesday.

Today's draft problem

To be presented by today's draftee

Let X be a geometric random variable. Give both an algebraic proof and a verbal explanation of the following:

$$P\{X = n + k \mid X > n\} = P\{X = k\}$$

If X is continuous random variable and g a continuous function, how can we determine PDF of g(X)?

Key: we need to understand the event $g(X) < y$.

Examples:

- (Ex 7a) X uniform on $(0,1)$ and $g(x) = x^n$
- (Ex 7b) X any continuous random variable with PDF f_X and $g(x) = x^2$
- (Ex 7c)X any continuous random variable with PDF f_X and $g(x) = |x|$

PDFs of (strictly monotone) functions of random variables

Theorem 7.1

Let X be a continuous random variable having probability density f_X. Suppose that $g(x)$ is a strictly monotonic function of x (this condition can be relaxed a little sometimes). Then the random variable $Y = g(X)$ has a probability density function given by

$$f_Y(y) = \begin{cases} f_X\left(g^{-1}(y)\right)\left|\frac{d}{dy}g^{-1}(y)\right|, & \text{if } y = g(x) \text{ for some } x \\ 0, & \text{else} \end{cases}$$

Let's prove this when g is decreasing (the increasing case is in the book).

Example

Sometimes we can use the previous Theorem even if g is not strictly monotone.

Example 7d: let X be a continuous, *nonnegative* random variable with density function f and let $g(x) = x^n$. Let's find PDF of $g(X)$.

Monday's draft problem

To be presented by Monday's draftee.

If X is uniformly distributed on $(0,1)$, find the density function of $Y = e^X$.