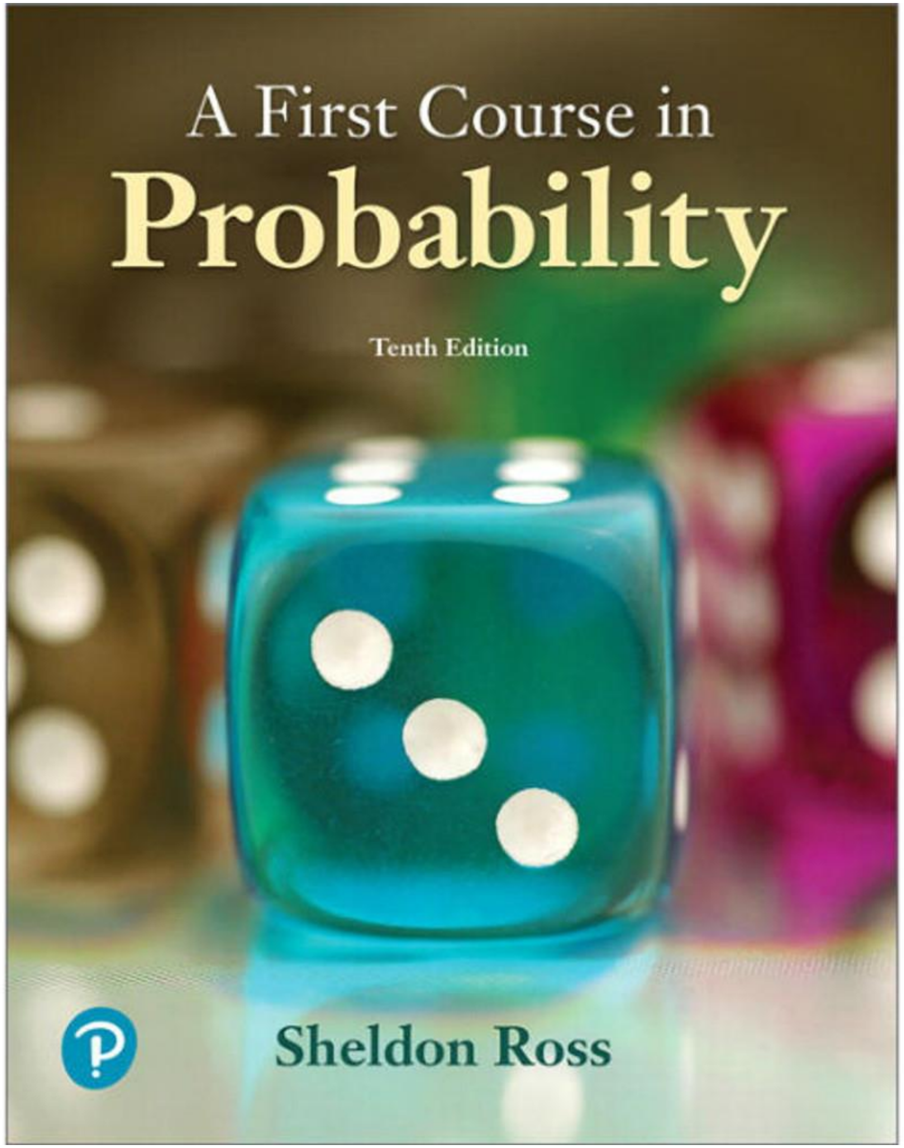
Lecture 13.1

Conditional distributions: discrete case

PURDUE UNIVERSITY | Department of Mathematics



Today's reading: 6.4

Next class: 6.5

HW10 now available. Due Friday, 4/18.

Remember: we have TWO draft problems every day now, an "A" problem and a "B" problem.

Friday's draft problem B

To be presented by today's B draftee.

Consider two independent random variables X_1 and X_2 such that X_1 follows an exponential distribution with mean 2, and X_2 follows a uniform distribution on the interval $[0,2\pi]$. Let $Y_1 = \sqrt{X_1}\cos X_2$ and $Y_2 = \sqrt{X_1}\sin X_2$.

1. Show that Y_1 and Y_2 are independent.
2. Show that Y_1 and Y_2 are both standard normal variables.

Today's draft problem A

To be presented by today's A draftee.

At Dr. Gesund's office, the waiting time T is modeled by an exponential random variable with mean 10 minutes. Today the office proposes the following deal: if your waiting time is less than 20 minutes, then you pay the full amount of your visit. Otherwise, you get reimbursed your waiting time minus 20. We call X the amount which is reimbursed by the office. Find the CDF of X. Then find the probability that you get reimbursed twice in 5 visits.

Today's draft problem B

To be presented by today's B draftee.

Let X and Y be independent binomial random variables with parameters (n, p) and (m, q), respectively. Compute the PMF of $X + Y$.

Question of the day

We’re going to answer the following question for several examples today:

If X and Y are both discrete random variables and we understand their ***joint*** distribution (more specifically: their joint PMF), then what do their *conditional distributions* (more precisely: conditional PMFs) look like?

(Of course, this question is also interesting when X and Y are continuous, but we’ll talk about that on Wednesday!)

Definitions

Conditional PMF (of X conditioned on Y)

$$p_{X|Y}(x|y) = P\{X = x|Y = y\} = \frac{P\{X = x, Y = y\}}{P\{Y = y\}} = \frac{p(x,y)}{p_Y(y)}$$

(assuming $p_Y(y) > 0$)

Conditional CDF (of X conditioned on Y)

$$F_{X|Y}(x|y) = P\{X \leq\ x|Y = y\} = \sum_{a \leq x} p_{X|Y}(a|y)$$

(assuming $p_Y(y) > 0$)

Conditional PMF when X and Y are independent?

Conditional PMF (of X conditioned on Y)

$$p_{X|Y}(x|y) = P\{X = x|Y = y\} = \frac{P\{X = x, Y = y\}}{P\{Y = y\}} = \frac{p(x, y)}{p_Y(y)}$$

(assuming $p_Y(y) > 0$)

If X and Y are independent, this simplifies drastically. Do you see how?

$$p_{X|Y}(x|y) = \frac{p(x, y)}{p_Y(y)} = \frac{p_X(x)p_Y(y)}{p_Y(y)} = p_X(x)$$

Example

Suppose that $p(x, y)$ is the PMF of jointly distributed random variables X and Y with

$$p(0,0) = 0.2 \qquad p(0,1) = 0.4 \qquad p(1,0) = 0.1 \qquad p(1,1) = 0.3$$

Compute all of the conditional PMFs. (How many are there??)

Example

If X and Y are independent Poisson random variables with parameters λ_X and λ_Y, respectively, then show the conditional distribution of X given that $X + Y = n$ is binomial. What are its parameters? (Hint: recall from last class that $X + Y$ is Poisson… with what parameter?)

Example

Suppose n independent trials are going to be performed, and each trial can result in one of k outcomes, with outcome $i = 1,2, \dots, k$ occurring with probability p_i. Let X_i be the number of trials that result in outcome i.

a) Find the joint PMF $p(X_1 = n_1, \dots, X_k = n_k)$. This is called a ***multinomial*** distribution.

b) Show that the conditional distribution of the X_i given that $X_{k-1} = n_{k-1}, X_k = n_k$ is another multinomial distribution.

Wednesday's draft problem A

To be presented by Wednesday's A draftee.

Choose a number X uniformly at random from the set of numbers $\{1,2,3,4,5\}$. Now chose a number at random from the subset no larger than X (that is, from $\{1, \dots, X\}$). Call this second number Y.

a) Find the joint PMF of X and Y
b) For each $i = 1,2,3,4,5$, find the conditional PMF of X given that $Y = i$.
c) Are X and Y independent?

Wednesday's draft problem B

To be presented by Wednesday's B draftee.

Suppose that a university sells 225 permits for a student parking lot with 190 spots, and the students are independent and identically distributed, wanting to park in the lot 80% of the time. Use the central limit theorem to compute the probability that at least one student won't be able to park in the lot. (Hint: use continuity correction).