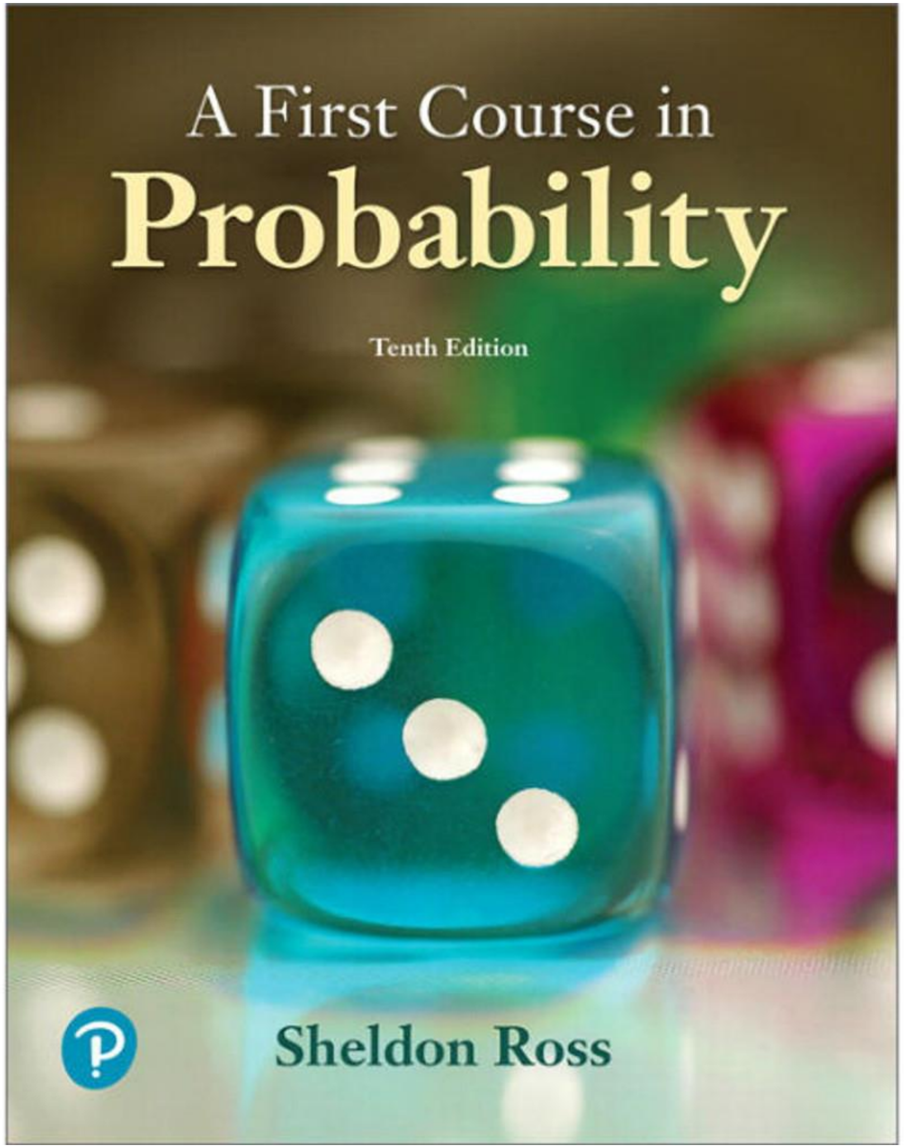
Lecture 13.3

Conditional distributions: continuous case

PURDUE UNIVERSITY | Department of Mathematics



Today's reading: 7.1, 7.2

Next class: 7.3

HW10 due now.

HW11 is your LAST homework assignment!
Available now, due 4/25

Wednesday’s draft problem B

Needs to be finished in Section 005…

Suppose that a university sells 225 permits for a student parking lot with 190 spots, and the students are independent and identically distributed, wanting to park in the lot 80% of the time. Use the central limit theorem to compute the probability that at least one student won’t be able to park in the lot. (Hint: use continuity correction).

Today’s draft problem A

To be presented by today’s A draftee.

The joint PDF of X and Y is

$$f(x,y) = f(x) = \begin{cases} c(x+y), & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

a) Find c

b) Find $f_{X|Y}(x|y)$ for $0 < y < 1$

Today's draft problem B

To be presented by today's B draftee.

Let X and Y be independent exponential random variables with the same rate λ. Find:

a) $f_{X|X+Y}(x|t)$

b) $F_{X|X+Y}(x|t)$

Have I annoyed you yet by repeating the phrase "EXPECTATION IS LINEAR" too many times?

Well, if so, I refuse to apologize ☺

Because LINEARITY OF EXPECATION is an extremely useful property! It holds ALWAYS, even if we take sums of random variables that are NOT independent!

If I haven't convinced you of the usefulness of this already, then I'm going to spend most of today giving you lots of little examples to try to seal the deal.

EXPECTATION IS LINEAR

For any $\mathbb{R}$-valued random variables X and Y, and any real numbers $a, b \in \mathbb{R}$,

$$E[aX + bY] = aE[X] + bE[Y].$$

A more general fact about expectation of functions of random variables

Proposition 2.1

Let $g(x, y)$ be any function $\mathbb{R}^2$ to $\mathbb{R}$.

If X and Y are jointly discrete with joint PMF $p(x, y)$, then

$$E[g(X,Y)] = \sum_{y} \sum_{x} g(x,y)p(x,y)$$

If X and Y are jointly continuous with joint PDF $f(x, y)$, then

$$E[g(X,Y)] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(x,y)f(x,y)dx\,dy$$

See book for proof in case where $g(x, y) \geq 0$ and X, Y jointly continuous.
Monday's B draft problem asks you to prove this when $g(x, y) \geq 0$ and X, Y jointly discrete

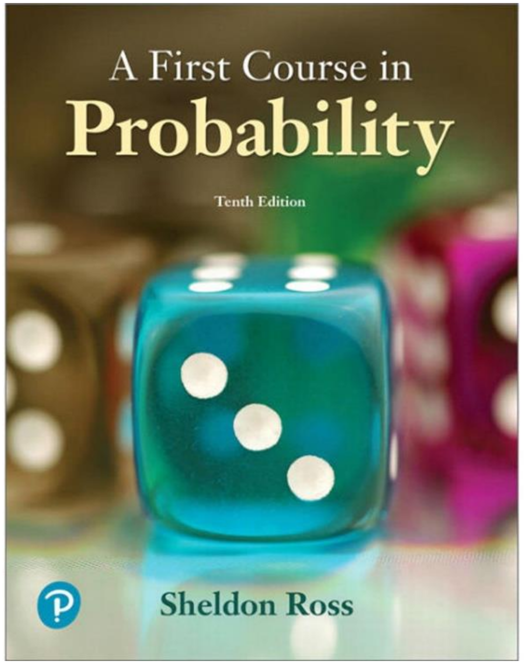
Examples

2c: mean of sample mean is… the mean!
2d: proving Boole’s inequality using indicator variables
2n: proving inclusion-exclusion identity using indicator random variables
2e: computing the mean of a binomial random variable using linearity of expectation
2h: what is the expected number of matches in the matching problem?
2i: if there are N Pokémon types, and we have an equal probability of seeing any type in a given encounter, how many encounters should we expect to need before we “catch ‘em all”? (assuming we always use Master Balls)

Monday's draft problem A

To be presented by Monday's A draftee.

Prove Proposition 2.1 (in Chapter 7) in the case that $g(x, y)$ is nonnegative and X and Y are jointly discrete.



Monday's draft problem B

To be presented by Monday's B draftee.

Let X and Y be random variables with joint probability mass function defined by:

	$Y=-1$	$Y=0$	$Y=2$	$Y=6$
$X=-2$	$3/27$	$1/27$	$1/27$	$3/27$
$X=1$	$6/27$	0	$3/27$	$3/27$
$X=3$	0	0	$3/27$	$4/27$

a) Compute $E[XY]$
b) Determine if X and Y are independent.