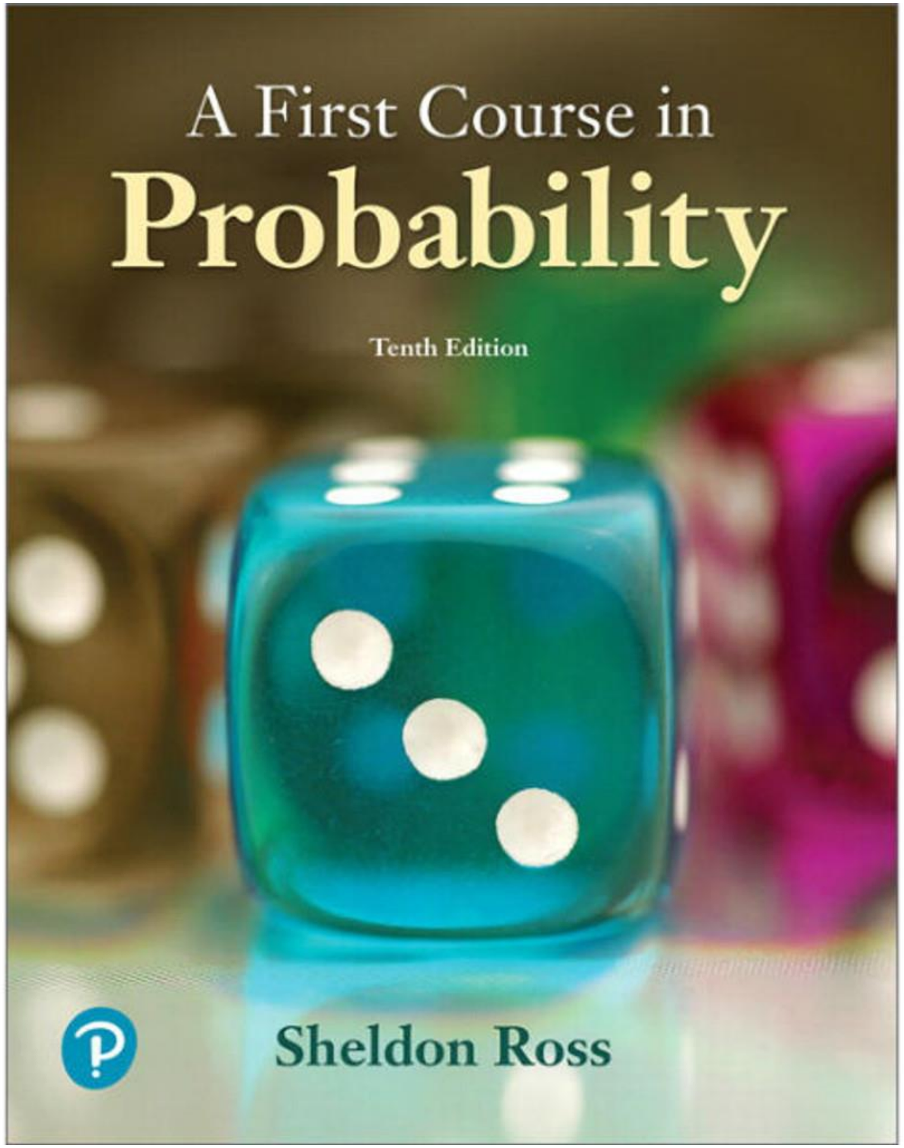
Lecture 2.1

Sample spaces

PURDUE UNIVERSITY
Department of Mathematics



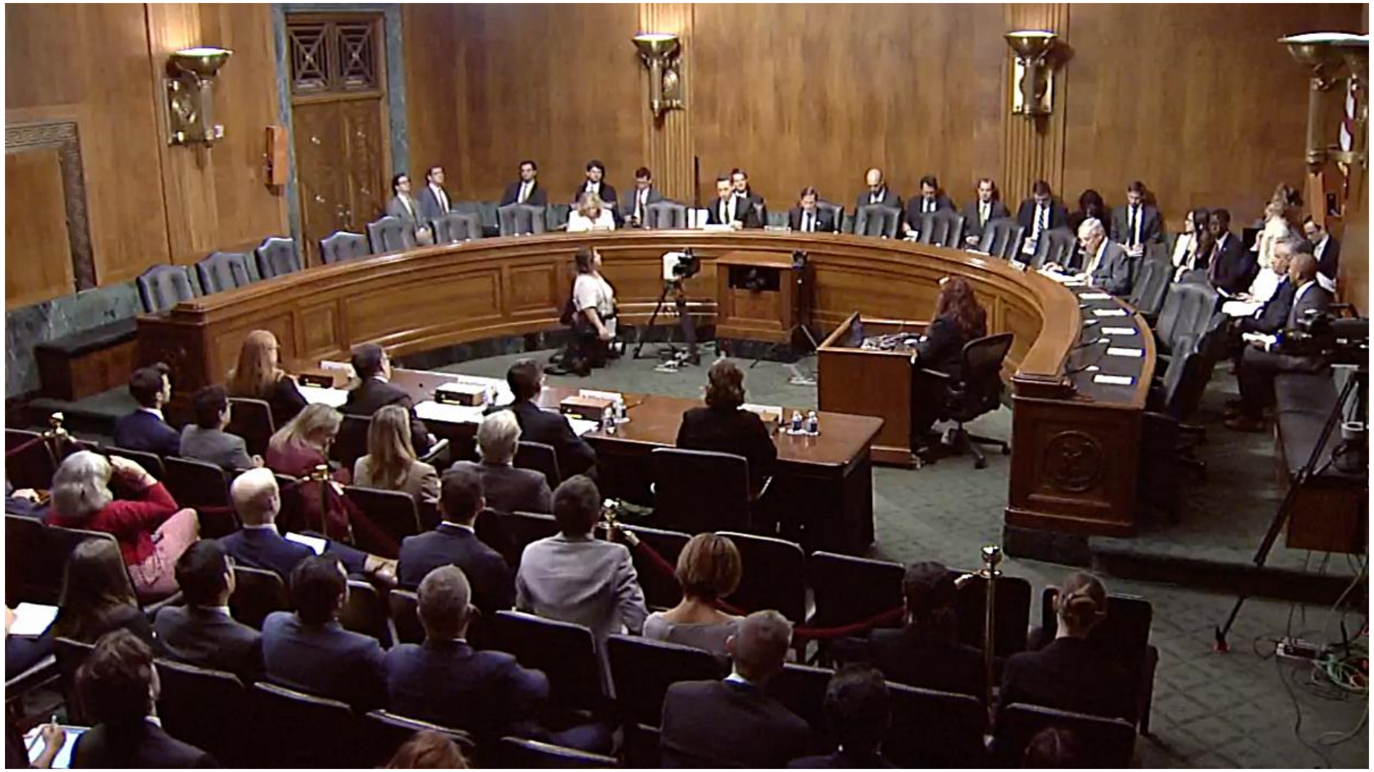
Today's reading: 2.1+2.2

Reading for next class: 2.3

MA/STAT 416 Help Room is <u>NOW OPEN!</u>

Today's draft problem

To be presented by today's draftee.



a) Give an algebraic proof that

$$\binom{n}{k}\binom{k}{j} = \binom{n}{j}\binom{n-j}{k-j}.$$

a) Now give a combinatorial proof of the same identity. (Hint: the picture on the left is of a Congressional subcommittee.)

Sample space

Definitions:

The set of all outcomes of an experiment is called the *sample space*. Usually denoted S.

Given a sample space S, an *event* is any subset E of S. denoted $E \subset S$.

Example: roulette

In roulette, the sample space is all possible places the ball can land after the wheel is spun.

An *event* is any set of such spaces, e.g.

$\{00, 0, 5\}$ or $\{all\ red\ spaces\}$ or $\{1\}$.

Example: roulette

In the rules of roulette, you don't have to bet on single spaces only. You are allowed to bet on various events. The roulette table shows these.

Notice: the table does not tell you the ODDS of different events. These are important though! They tell you how how likely different events are after a spin. They also determine the payout in a win.

However: We are NOT discussing this today!

“Boolean algebra:” the algebra of events

Because the events E determined by a sample space S are exactly the same thing as the subsets of S, any of the usual ways we might combine subsets of S can be used to describe events too.

Let $E \subset S$ and $F \subset S$ be two events. The event $E \cup F \subset S$ is called the *union* of E and F, and is defined as the set of all outcomes in S that are in either E **or** F.

Let $E \subset S$ and $F \subset S$ be two events. The event $EF \subset S$ is called the *intersection* of E and F, and is defined as the set of all outcomes in S that are in both E **and** F.

"Boolean algebra:" the algebra of events

Because the events E determined by a sample space S are exactly the same thing as the subsets of S, any of the usual ways we might combine subsets of S can be used to describe events too.

Let $E \subset S$ be an event. The event $\mathrm{E}^{\mathrm{c}} \subset S$ is called the *complement* of E, and is defined as the set of all outcomes in S that are **not** in E.

“Boolean algebra:” the algebra of events

Because the events E determined by a sample space S are exactly the same thing as the subsets of S, any of the usual ways we might combine subsets of S can be used to describe events too.

The empty set $\emptyset \subset S$ is called the *null event*.

If $EF = \emptyset$, then we say E and F are *mutually exclusive.*

Notice that E and E^c are ALWAYS mutually exclusive.

"Boolean algebra:" the algebra of events

We can take unions of more than just two events:

Let $E_1, E_2, E_3, \dots, \subset S$ be (infinitely many) events. The union

$$\bigcup_{i=1}^{\infty} E_i \subset S$$

is defined as the event consisting of all outcomes in *at least one* of the events E_i (for some $i = 1,2,3,\dots$).

"Boolean algebra:" the algebra of events

Similarly

Let $E_1, E_2, E_3, \dots, \subset S$ be (infinitely many) events. The intersection

$$\bigcap_{i=1}^{\infty} E_i \subset S$$

is defined as the event consisting of all outcomes in *all* of the events E_i (for all $i = 1,2,3,\dots$).

“Boolean algebra:” the algebra of events

Similarly

Let $E_1, E_2, E_3, \dots, \subset S$ be (infinitely many) events. The intersection

$$\bigcap_{i=1}^{\infty} E_i \subset S$$

is defined as the event consisting of all outcomes in *all* of the events E_i (for all $i = 1,2,3,\dots$).

Important for intepretation

Suppose $E \subset F \subset S$. This means the following: if event E happens, then so does event F.

Basic algebraic laws for manipulating events

Commutative laws

$$E \cup F = F \cup E \qquad EF = FE$$

Associate laws

$$(E \cup F) \cup G = E \cup (F \cup G) \qquad (EF)G = E(FG)$$

Distributive laws

$$(E \cup F)G = EG \cup FG \qquad (EF) \cup G = (E \cup G)(F \cup G)$$

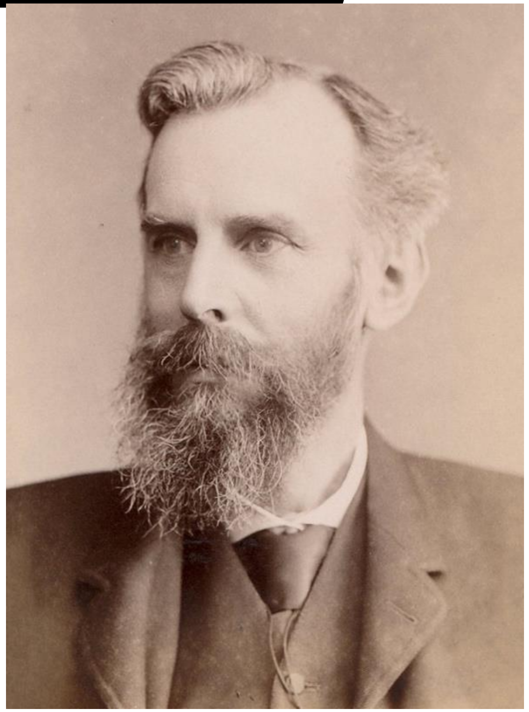
De Morgan's laws

$$\left(\bigcup_{i=1}^{\infty} E_i\right)^c = \bigcap_{i=1}^{\infty} E_i^c$$

$$\left(\bigcap_{i=1}^{\infty} E_i\right)^c = \bigcup_{i=1}^{\infty} E_i^c$$

Friday's draft problem

To be presented by the draftee at the beginning of class on Friday.



Using Venn diagrams:

a) Simplify the expression $(E \cup F)(E \cup F^c)$.

b) Show $(E \cup F)^c = E^c F^c$.

c) Show $(EF)^c = E^c \cup F^c$.