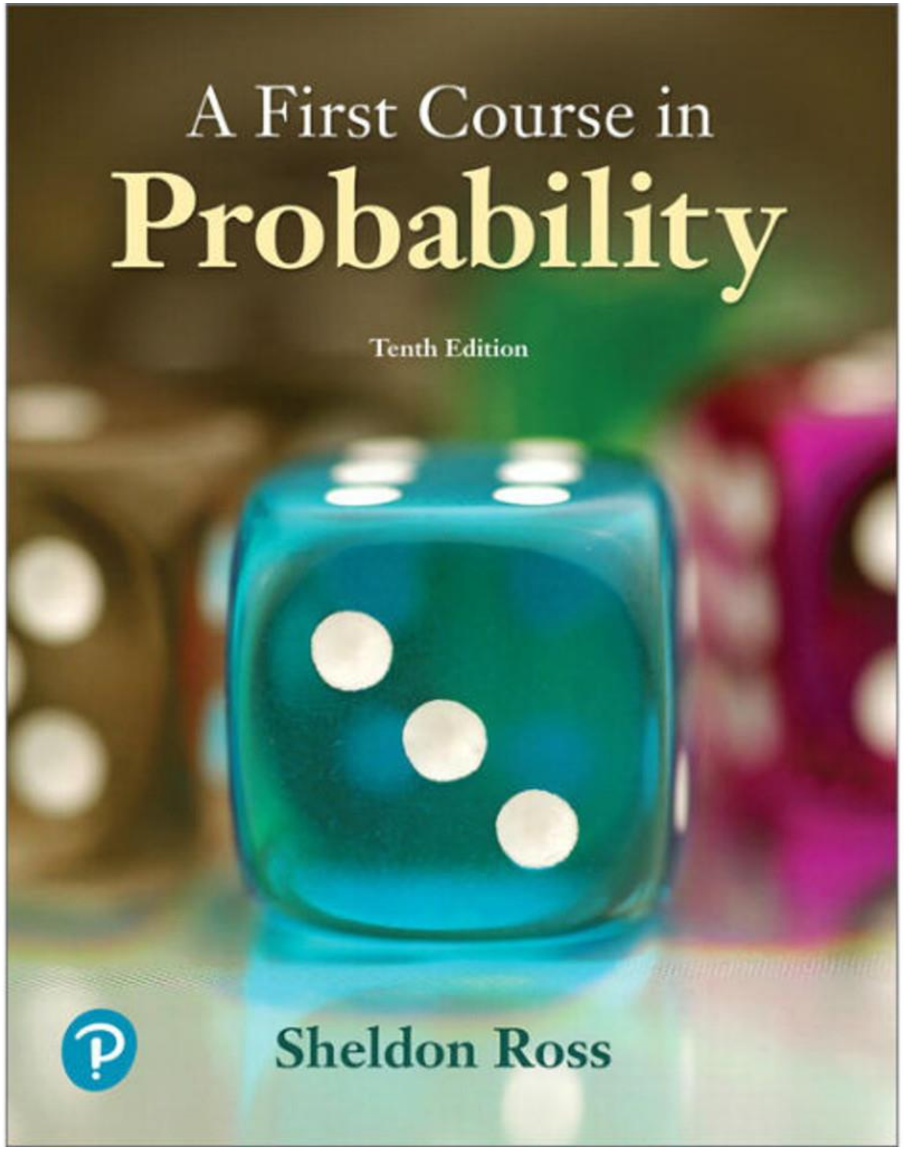
Lecture 3.3

Conditional probability

PURDUE UNIVERSITY | Department of Mathematics



Today's reading: 3.1+3.2

Reading for next class: 3.3

HW2 is due TODAY.

HW3 is now available, and due next Friday at the beginning of class.

Notes from the grader concerning HW:

1. If your homework has multiple pages, STAPLE IT!
2. Please write your section number (006 or 005) and your name at the top of your submission.
3. Do not use the wrong book!

Today’s draft problem

To be presented by today’s draftee.

Suppose that you must uniformly randomly draw 6 marbles from an urn that contains 7 red marbles, 3 green marbles, and 5 blue marbles. What is the probability that you draw 3 red marbles, 2 green marbles and 1 blue marble?

Conditional probability

Let P be a probability measure on a sample space S, and let E and F be two events in S.

Definition
If $P(F) > 0$, then the *conditional probability* of E given that F has occurred is the number $P(E|F)$ defined as

$$P(E|F) = \frac{P(EF)}{P(F)}$$

Note that we need $P(F) > 0$ in order to be able to divide by it!

Conditional probability - interpretation

The conditional probability

$$P(E|F) = \frac{P(EF)}{P(F)}$$

is the correct way to assign a probability to the event E if we are *already certain* that event F has occurred.

(We will show next week that $P(-|F)$ gives a new probability measure on the sample space S.)

Conditional probability - motivations

1. If we learn new information, then we should update our probabilities. Conditional probabilities are the correct way to do this.
2. Sometimes, we can use conditional probabilities to compute *unconditional* probabilities

Bayes's theorem (next week) will allow us to make this point more precise.

I'm not saying anything deep here! Simply: if $P(E|F) = P(EF)/P(F)$, then $P(EF) = P(E|F)P(F)$.

Conditional probability - multiplication rule

More generally, we have:

$$P(E_1 E_2 \cdots E_n) = P(E_1)P(E_2|E_1)P(E_3|E_1 E_2) \cdots P(E_n|E_1 \cdots E_{n-1})$$

conditional probabilities to compute *unconditional* probabilities

I'm not saying anything deep here! Simply: if $P(E|F) = P(EF)/P(F)$, then $P(EF) = P(E|F)P(F)$.

Monday’s draft problem

To be presented by Monday’s draftee at the beginning of class.

Nine cards are randomly selected (without replacement) from a standard deck of 52 playing cards in the following way: first, two are given to your opponent, then two are given to you, and then five are shown face-up.

Determine the conditional probability that one of the first two cards (dealt to your opponent) is a 5, given that your two cards are a 6 and a Q, and the last five cards have face values of 2, 3, 4, 6 and J.