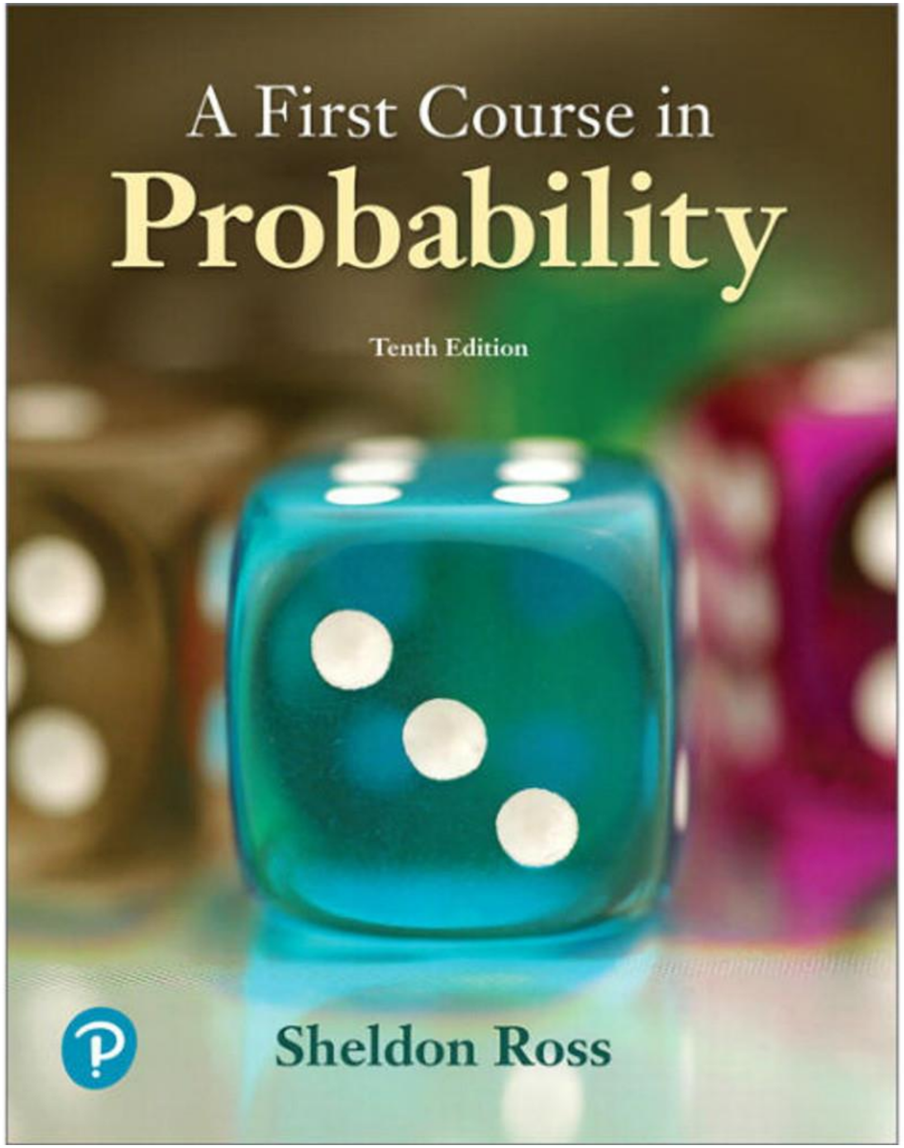
Lecture 4.3

Odds; independent events

Department of Mathematics



Today’s reading: 3.4

Reading for next class: continuing out of 3.4.

For Wednesday’s class: 3.5

MA/STAT 416 Help Room is <u>NOW OPEN!</u>

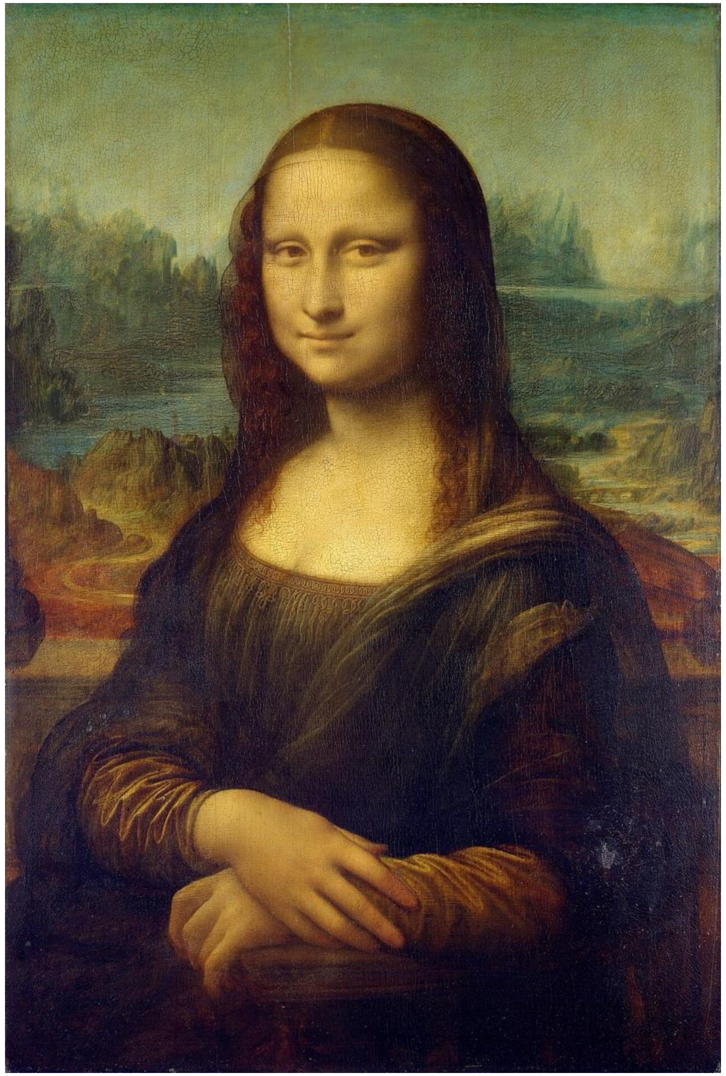
HW3 is due today.

HW4 is now available and due a week from now.

Today’s draft problem

To be presented by today’s draftee.

An expensive painting is stolen. ICE knows that the thieves have hidden it on one of four container ships that is coming into the port of Hamburg, but they don’t know which one, so they assume all four possibilities are equally likely. If the painting is in fact on the i^{th} ship, then the probability that ICE will find it is $1-\beta_i$ (i=1,2,3,4) for some $0<\beta_i<1$ (the “overlook probability”). For each i=1,2,3,4, determine the conditional probability that the painting is on ship i given that ICE searches ship 1 and finds nothing.



Odds

From the book:

Definition
If P is any probability measure on a sample space S and A is an event in S, then the ***odds*** of A are

$$\frac{P(A)}{P(A^c)} = \frac{P(A)}{1 - P(A)}$$

The odds of A tells us how much more likely it is that A occurs than it does not occur.

Odds version of easy Bayes's theorem

$$\frac{P(H|E)}{P(H^c|E)} = \frac{P(E|H)}{P(E|H^c)} \frac{P(H)}{P(H^c)}$$

New odds of hypothesis given new evidence

Original odds of hypothesis

In light of the new evidence, we should update our original probability of the hypothesis by multiplying by this "odds Bayes factor"

Proof & example on chalkboard

Independent events – intuitive idea

If we have two events E and F (in a sample space S with probability measure P), then, intuitively, when we say that E is "independent" of F, we mean that the likelihood of E occurring is unaffected by whether or not F occurs.

We can make this mathematically precise using conditional probabilities: E is independent of F if

$$P(E|F) = P(E)$$

Independent events – definition for two events

However, we can express this more simply, since, by definition of conditional probability, $P(E|F) = P(EF)/P(F)$. Thus,

$$P(E|F) = P(E)$$

if and only if

$$P(EF) = P(E)P(F).$$

Note that this is symmetric in E and F!

So, we make the following definition:

E is *independent* of F if $P(EF) = P(E)P(F)$. If E is not independent of F, then we say E and F are *dependent*.

Example on chalkboard

Complements & independence

From the book:

Proposition 4.1.
If E and F are independent events, then E and F^c are independent. (My addition: hence, so are E^c and F^c, and E^c and $\mathrm{F}.$)

Proof on chalkboard

Independent events – general definition

Three events E, F and G are *independent* if

$$P(EFG) = P(E)P(F)P(G)$$
$$P(EF) = P(E)P(F)$$
$$P(EG) = P(E)P(G)$$
$$P(FG) = P(F)P(G)$$

Several events $E_1, E_2, \ldots, E_k$ are *independent* if

$$P\left(E_{i_1}E_{i_2}\cdots E_{i_r}\right) = P\left(E_{i_1}\right)P\left(E_{i_2}\right)\cdots P\left(E_{i_r}\right)$$

for every $r = 1,2,\ldots,k$ and every choice of r of the k events.

[Check in with your understanding: how many conditions are there here?]

Monday's draft problem

To be presented by Monday's draftee at the beginning of class.

There are 150 types of Pokémon. When wandering around Kanto, the probability that Ash Ketchum has an encounter with Pokémon i is p_i (where $\sum_{i=1}^{150} p_i = 1$, of course).

Assume that Ash has N encounters with Pokémon. Let A_i be the event that at least one of these encounters is with Pokémon i. Let i, j, k be 3 distinct Pokémon. Determine $P(A_i | A_j \cup A_k)$.