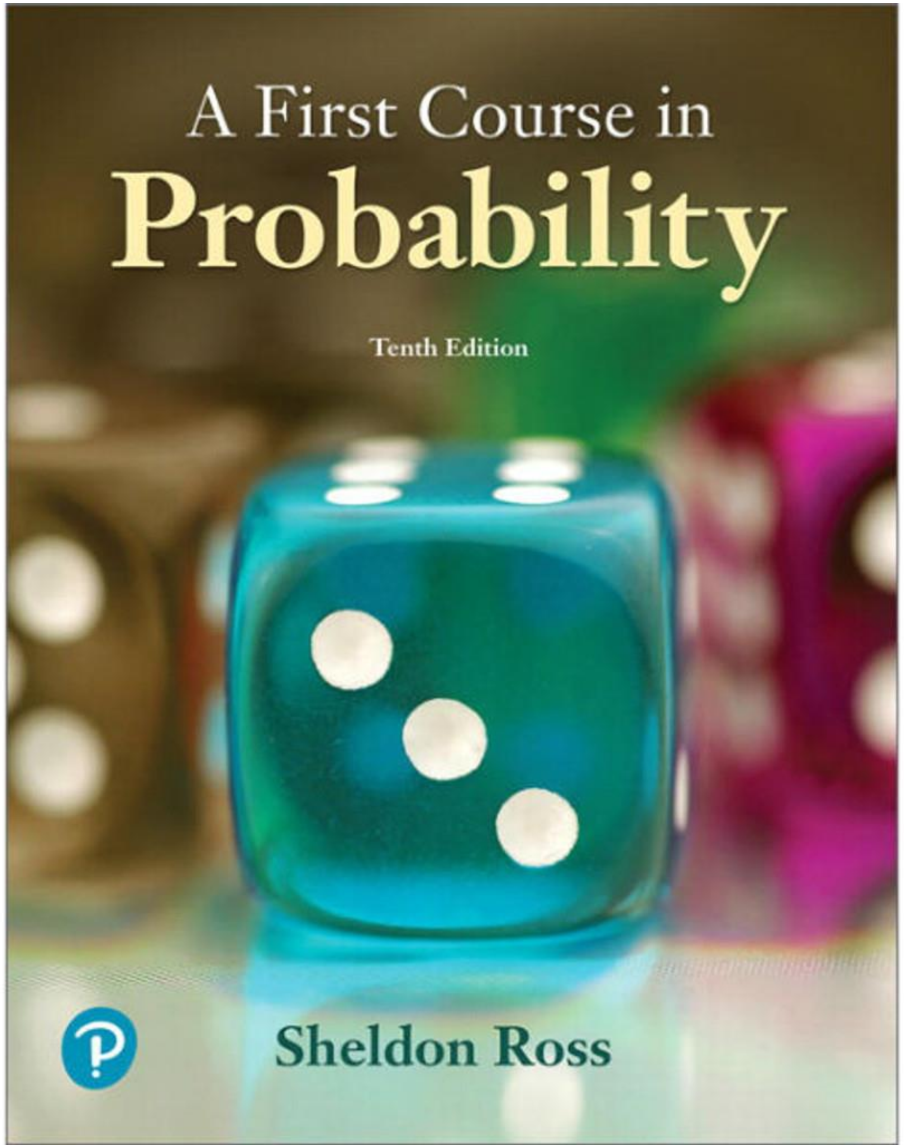
Lecture 7.1

Expectation value

PURDUE UNIVERSITY | Department of Mathematics



Today's reading: 4.3 and 4.4

Next class: examples from 4.3 and 4.4

HW5 is due Friday at beginning of class.

Midterm Exam 1 Class Statistics

Number of submitted grades: 85 / 85

Minimum: 20 %

Maximum: 100 %

Average: 72.79 %

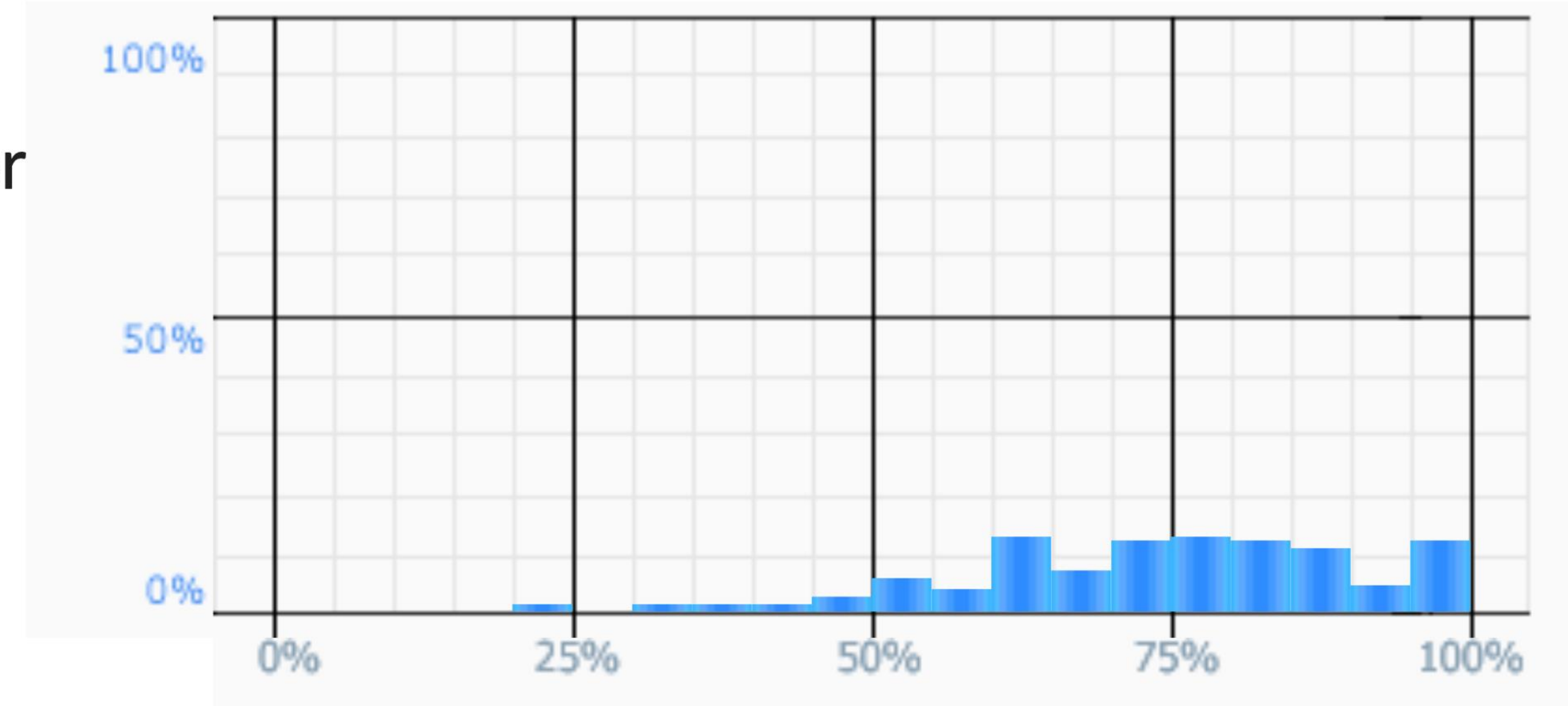
Mode: 75 %, 85 %, 62.5 %

Median: 75 %

Standard Deviation: 16.51 %

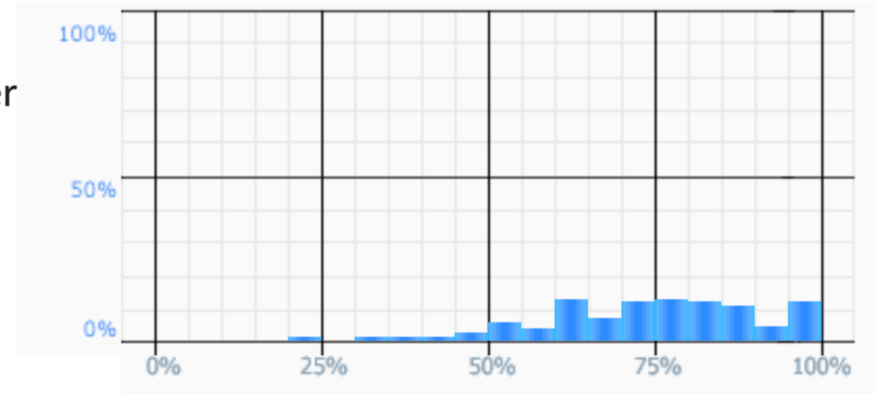
Grade Distribution

Number
of
Users
(%)



Grade Distribution

Number of Users (%)



- If you got at least 30/40 on the exam, then you should feel good about the prospects of earning an A or B at the end of the semester.

- If you got 20/40 or less, then you will need to work hard to get a B. An A is possible, but will also, frankly, probably require some luck.

- If you got less than 16/20, then your likelihood of getting an A this semester is near 0; even getting a B is going to require hard work to seriously turn things around and learn from your mistakes (in a way that leads to significantly higher scores on MT2 and the final).

Wednesday's draft problem

To be presented by Wednesday's draftee.

4. [10 points] There are 150 types of Pokémon. Ash Ketchum repeatedly has encounters with them. In any one encounter, the probability that it is a Pokémon of type i is p_i (where $\sum_{i=1}^{150} p_i = 1$), and all encounters are independent events. Let $N \geq 1$. What is the probability that in N encounters, the Pokémon Ash encounters last (*i.e.* in the N^{th} encounter) is a new type of Pokémon that he did not see in any of the previous $N-1$ encounters?

Hint: let E be the event that on the N^{th} encounter, Ash sees a new type of Pokémon.
Let E_i be the event that on the N^{th} encounter, Ash sees a new type of Pokémon and it is of type i.

Today's draft problem

To be presented by today's draftee.

5 Boilermakers and 5 Hoosiers are to be ranked by how cool they are (according to a Buzzfeed quiz). Assume that each of the 10! possible rankings is equally likely. Let X be the highest ranking achieved by a Boilermaker. (For instance, X=1 if the highest ranked person is a Boilermaker.). Find P{X=i} for each i=1,2,…,9,10.

$$\overset{?}{\geq}$$

Expected value (AKA mean, AKA average)

Definition (from the book, more-or-less):

Let $X: S \to \mathbb{R}$ be a (real-valued) random variable with PMF $p: \mathbb{R} \to \mathbb{R}$ (in particular, X must be discrete). The *expected value of* X is

$$E[X] = \sum_{x \in \mathbb{R} \,:\, p(x) > 0} x p(x)$$

Also called the "mean" of X, or the "average" of X, or the "first moment of X" (more on this last one later).

Expected value - small examples

Variation of Ex. 3a:

If X is the sum of the result when rolling two standard dice, what is $E[X]$?

Ex. 3b:

We $I : S \to \mathbb{R}$ is an *indicator variable* for the event $A \subset S$ if

$$I(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases}$$

What is $E[I]$?

Functions of a random variable

Definition (not really in the book):

Let $X: S \to \mathbb{R}$ be a random variable. Any function $g: \mathbb{R} \to \mathbb{R}$ can be understood as a "function of X." More precisely: we can compose X and g to get a new random variable

$$\begin{aligned} g(X) = \ g \circ X: S &\to \mathbb{R} \\ a &\mapsto g(X(a)) \end{aligned}$$

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Expectation of a function of a random variable

Proposition 4.1 (more-or-less from book):

Let $X: S \to \mathbb{R}$ be a discrete random variable that takes values $x_1, x_2, x_3, \ldots,$ and has PMF $p(x)$. Then for any $g: \mathbb{R} \to \mathbb{R}$

$$E[g(X)] = \sum_{i=1}^{\infty} g(x_i)p(x_i)$$

Proofs on chalkboard

Corollary 4.1 (more-or-less from book):

Let $X: S \to \mathbb{R}$ be a discrete random variable that takes values $x_1, x_2, x_3, \ldots,$ and has PMF $p(x)$. Then for any real numbers $a, b \in \mathbb{R}$,

$$E[aX + b] = aE[X] + b$$

Moments

Definition (from book):

Let $X: S \to \mathbb{R}$ be a discrete random variable. The n^{th} *moment* of X is $E[X^n]$.

From Proposition 4.1, it follows that if the PMF of X is $p(x)$, then

$$E[X^n] = \sum_{x:\, p(x)>0} x^n p(x)$$