Lecture 7.2

More examples w/ expectation value

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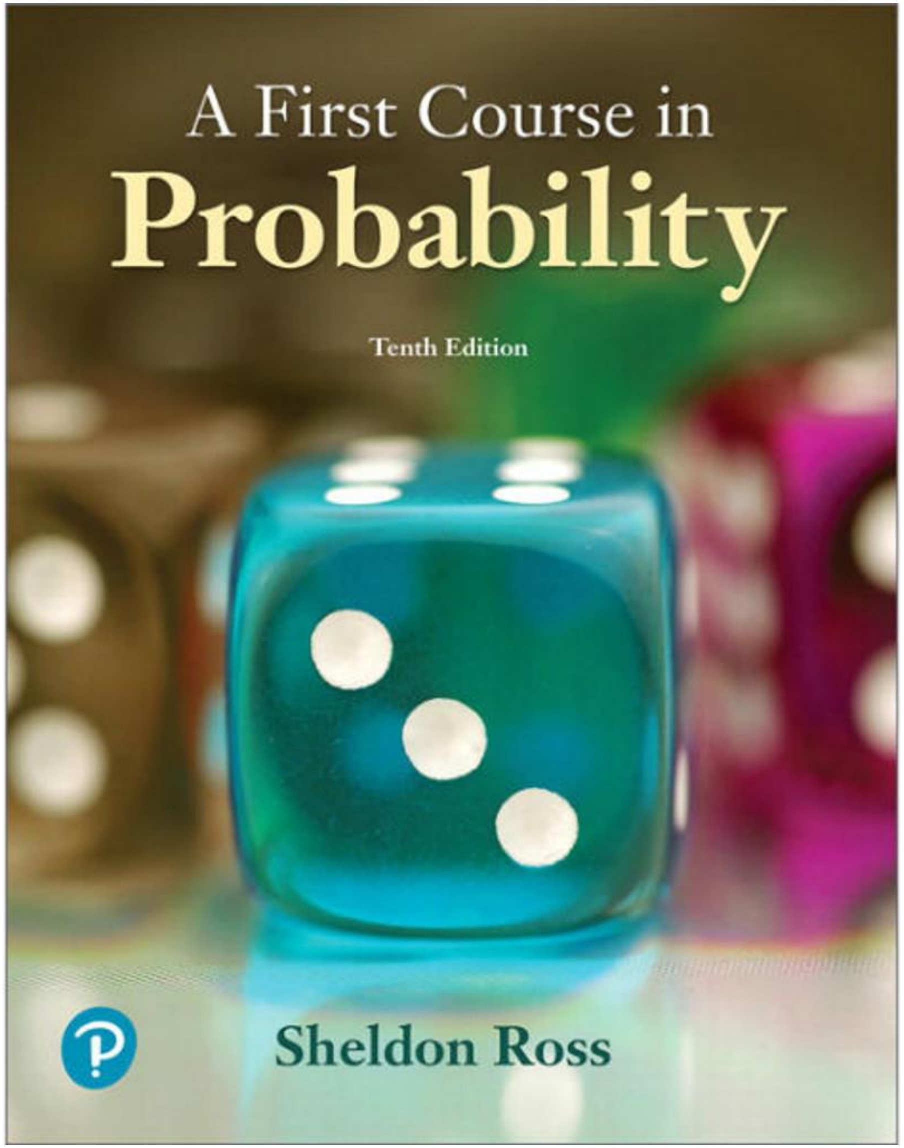

Today's reading: continuing out of 4.3 and 4.4

Next class: 4.5

HW5 is due Friday at beginning of class.

I have updated a few MT1 scores after talking with some of you in my office hours. If you think you deserve points back, then please come talk to me!

Today's draft problem

To be presented by today's draftee.

4. [10 points] There are 150 types of Pokémon. Ash Ketchum repeatedly has encounters with them. In any one encounter, the probability that it is a Pokémon of type i is p_i (where $\sum_{i=1}^{150} p_i = 1$), and all encounters are independent events. Let $N \geq 1$. What is the probability that in N encounters, the Pokémon Ash encounters last (*i.e.* in the N^{th} encounter) is a new type of Pokémon that he did not see in any of the previous $N-1$ encounters?

Hint: let E be the event that on the N^{th} encounter, Ash sees a new type of Pokémon.
Let E_i be the event that on the N^{th} encounter, Ash sees a new type of Pokémon and it is of type i.

Recall - Expected value

Definition (from the book, more-or-less):

Let $X: S \to \mathbb{R}$ be a (real-valued) random variable with PMF $p: \mathbb{R} \to \mathbb{R}$ (in particular, X must be discrete). The *expected value of* X is

$$E[X] = \sum_{x \in \mathbb{R} \,:\, p(x) > 0} xp(x)$$

Also called the "mean" of X, or the "average" of X, or the "first moment of X" (more on this last one later).

Recall - Expectation of a function of a random variable

Proposition 4.1 (more-or-less from book):

Let $X: S \to \mathbb{R}$ be a discrete random variable that takes values $x_1, x_2, x_3, \ldots,$ and has PMF $p(x)$. Then for any $g: \mathbb{R} \to \mathbb{R}$

$$E[g(X)] = \sum_{i=1}^{\infty} g(x_i)p(x_i)$$

006: I owe you the proof of the Corollary

005: I owe you the proof of the Proposition

Corollary 4.1 (more-or-less from book):

Let $X: S \to \mathbb{R}$ be a discrete random variable that takes values $x_1, x_2, x_3, \ldots,$ and has PMF $p(x)$. Then for any real numbers $\mathrm{a}, \mathrm{b} \in \mathbb{R}$,

$$E[aX + b] = aE[X] + b$$

Moments

Definition (from book):

Let $X: S \to \mathbb{R}$ be a discrete random variable. The n^{th} *moment* of X is $E[X^n]$.

From Proposition 4.1, it follows that if the PMF of X is $p(x)$, then

$$E[X^n] = \sum_{x:\, p(x)>0} x^n p(x)$$

Friday's draft problem

To be presented by Friday's draftee.

Fix a positive integer k. Purdue and IU are going to play a series of sportsball games until one of them wins k of them. The probability that Purdue wins any one game is $p \geq 0.5$, and the results of different games are independent of one another.

(a) If $k = 3$, find the expected number of games played.

(b) What is the value of p that maximizes the expected number of games played? (You may assume $k = 3$ if it makes it easier.)