Lecture 7.3

Variance

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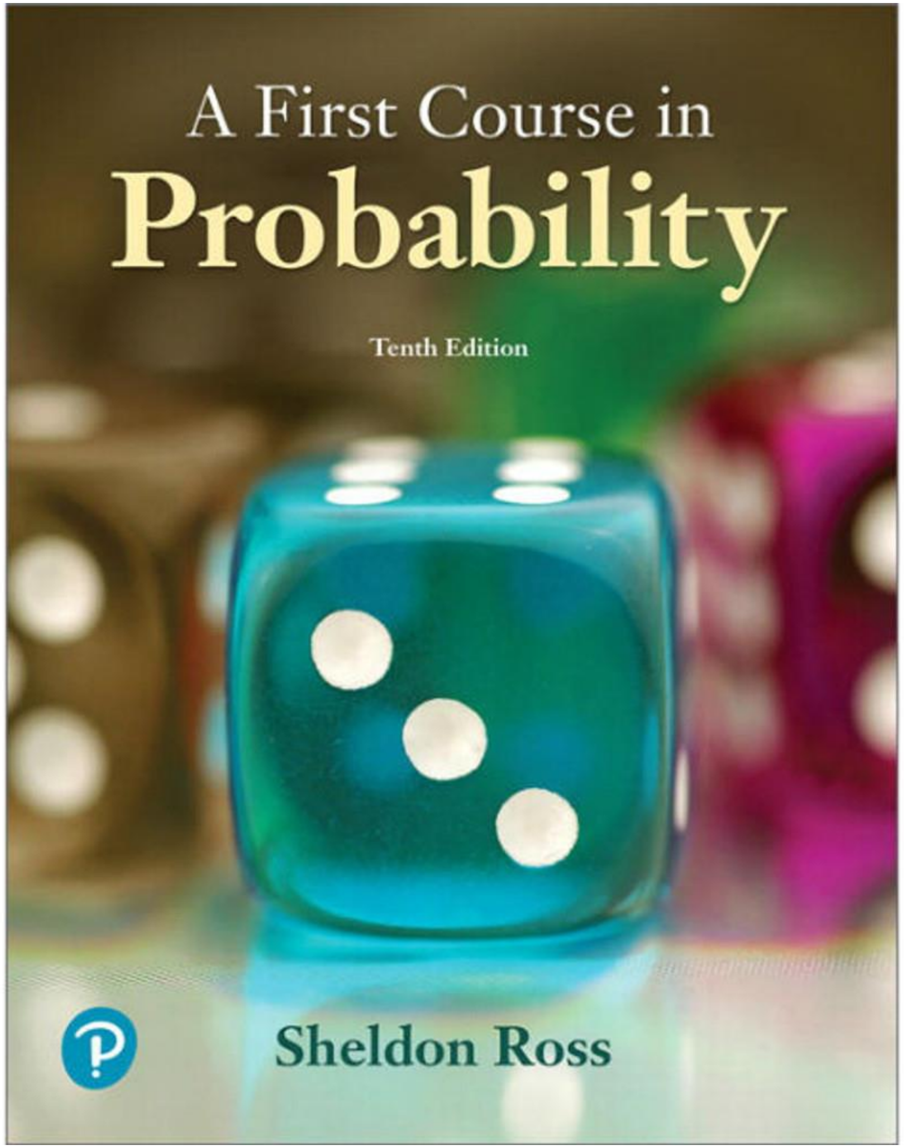

Today's reading: 4.5

Next class: 4.6

HW5 due now.

HW6 now available

I have updated a few MT1 scores after talking with some of you in my office hours. If you think you deserve points back, then please come talk to me!

Today's draft problem

To be presented by today's draftee.

Fix a positive integer k. Purdue and IU are going to play a series of sportsball games until one of them wins k of them. The probability that Purdue wins any one game is $p \geq 0.5$, and the results of different games are independent of one another.

(a) If $k = 3$, find the expected number of games played.

(b) What is the value of p that maximizes the expected number of games played? (You may assume $k = 3$ if it makes it easier.)

Variance

Definition (from the book, more-or-less):

Let $X: S \to \mathbb{R}$ be a (discrete) random variable with expectation value/mean $\mathrm{E}[X] = \mu$. The <u>variance</u> of X is defined to be

$$Var(X) = E[(X - \mu)^2].$$

In fact, it turns out that

$$Var(X) = E[X^2] - (E[X])^2$$

[Proof on chalkboard.]

Intuition

- The mean/expectation value of a random variable is the "center of mass" of its PMF.
- The variance quantifies the "spread" of the PMF around its center of mass. (More precisely, it is like the "moment of inertia" in mechanics.)

Two random variables/PMFs can have the same mean but different variance. Can you think of an example?

A couple of useful identities for variance

It also turns out that

$$Var(X) = E[(X-\mu)^2] \geq 0$$

[Why?]

Since

$$Var(X) = E[X^2] - (E[X])^2,$$

this implies

$$E[X^2] \geq (E[X])^2$$

The following is also sometimes helpful:

$$Var(aX+b) = a^2 Var(X)$$

Why is this true? How does this compare to the behavior of $E[aX+b]$?

Standard deviation

Definition:

If X is a random variable with variance $\mathrm{Var}(X)$, then the standard deviation of X is defined to be

$$SD(X) = \sqrt{Var(X)}$$

We won't do much with this now, but it will be useful later.

Monday's draft problem

To be presented by Monday's draftee.

Fix a positive integer k. Purdue and IU are going to play a series of sportsball games until one of them wins k of them. The probability that Purdue wins any one game is $p \geq 0.5$, and the results of different games are independent of one another.

(a) Let X be the random variable that counts the number of total games played. If $k = 2$, find the variance of X.

(b) What is the value of p that maximizes this variance? (Continue to assume $k = 2$.)