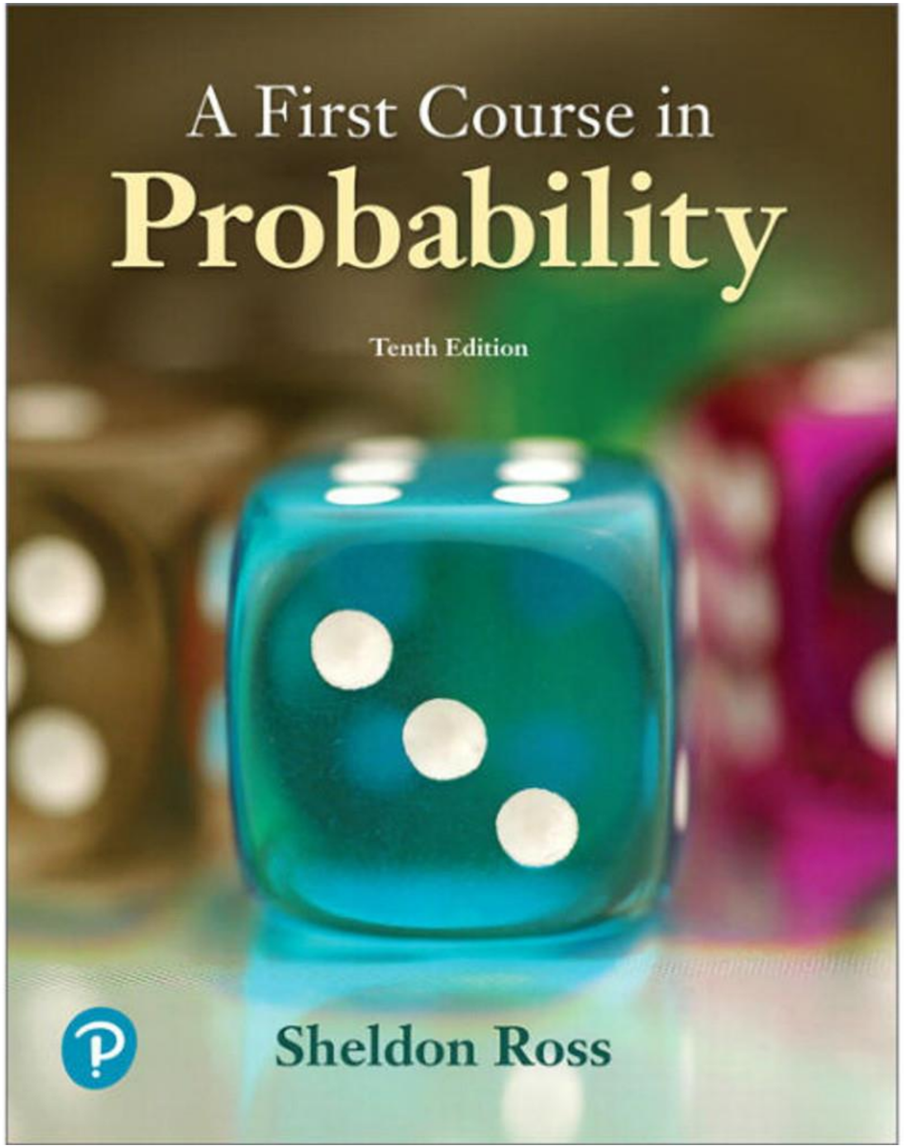
Lecture 8.2

Poisson Random Variables

PURDUE UNIVERSITY | Department of Mathematics



Today’s reading: 4.7

Next class: 4.8

HW6 now available

Note: office hour schedule change

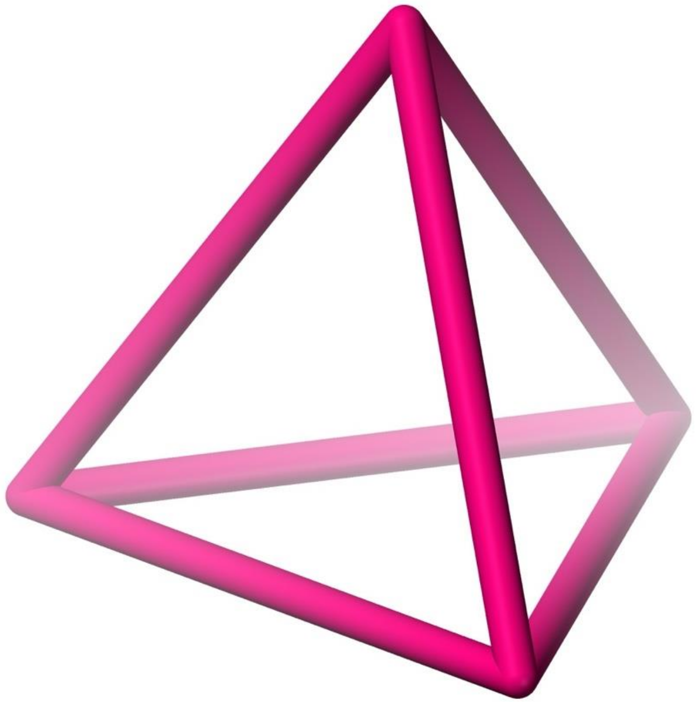
- Thursday → Friday 10-11am (still virtual)
- Friday → 12:30-1:30 (in person)

Today's draft problem

To be presented by today's draftee.

This is a variation of Example 6c in Chapter 4 of Ross.

Three, fair, **four-sided** dice (number 1,2,3,4) are to be tossed, and a player bets on one of the numbers 1 through 4. If the number bet by the player appears i times, $i = 1,2,3$, then the player wins i dollars; if the number bet by the player does not appear, then they lose 1 dollar. Should the player play? (That is, if they do play, should they expect to make money or lose money?)



Poisson random variables

Definition (from the book):

A *Poisson random variable* with parameter λ is a random variable X that takes the values $0, 1, 2, \ldots$ and has PMF given by

$$p(i) = P\{X = i\} = e^{-\lambda}\frac{\lambda^i}{i!} \qquad\qquad i = 0, 1, 2, \ldots$$

Why is this a valid PMF?

Expectation and variance of Poisson random variables

If X is a Poisson random variable with parameter λ then

$$E[X] = \lambda$$

and

$$Var(X) = \lambda$$

Why? [Be emotionally prepared: we will need to use the fact $e^{\lambda} = \sum_{i=0}^{\infty} \frac{\lambda^i}{i!}$]

Poisson random variables - WHY?!

Two explanations:

1. If X is a binomial random variable with parameters (n, p) where n is "large" and p is "small enough" so that $\mathrm{E}[X] = np$ is "not large," then a Poisson random variable with parameter $\lambda = np$ is a "good approximation" to X.

2. Poisson PMFs show up in situations where we want to count the number of times a relatively rare discrete event occurs in a large, continuous interval of time.

We'll make this first thing precise momentarily. We'll make the second thing precise on Friday.

Friday's draft problem

To be presented by Friday's draftee.

Suppose that the number of accidents that occur on a highway each day is a Poisson random variable X with parameter $\lambda = 3$.

(a) Find the probability that 3 or more accidents occur today.

(b) Repeat part (a) under the assumption that at least 1 accident occurs. (In other words, compute the conditional probability $P(\{X \geq 3\}|\{X \geq 1\})$.